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Modelling and analysis of a harvested prey–predator system incorporating a prey refuge

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Abstract

The present paper deals with a prey–predator model incorporating a prey-refuge and independent harvesting in either species. Our study shows that, using the harvesting efforts as controls, it is possible to break the cyclic behaviour of the system and drive it to a required state. The possibility of existence of bionomic equilibria has been considered. The problem of optimal harvest policy is then solved by using Pontryagin's maximal principle.

MSC: 34C05; 92D25

Keywords: Prey–predator; Prey-refuge; Limit cycles; Bionomic equilibrium; Optimal harvesting

1. Introduction

Economic progress and ecological balance always have conflicting interests. Catering to the necessities and comforts of human beings invariably robs the ecological structure of the nature. This, more often that not, leads to the extinction of a species of life. Often it is possible to prevent such extinction by proper planning. Such a planning has to be either by force or dissentive. For example, if a particular activity by individuals of a region is causing severe damage of the ecosystem of that region and if the activity is inevitable then the governing authority of the region should plan a regulating policy which would keep the damage of the ecosystem minimal. One such activity is harvesting, which has a strong impact on the dynamic evolution of a population subjected to it. Reasonable harvesting policies is indisputably one of

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the major and interesting problems from ecological and economical point of view. The exploitation of biological resources and harvest of population species are commonly practiced in fishery, forestry and wildlife management. A management of multispecies fisheries which is needed to maintain an ecological balance, that is disrupted due to over exploitation of many conventional fish stocks and growing interest in harvesting new kinds of food from the sea.

The problem of predator–prey interactions under constant rate of harvesting or constant quota of harvesting of either species or both species simultaneously have been studied by some authors. For example, Brauer and Soudak [2–5] studied a class of predator–prey models under constant rate of harvesting and under constant quota of harvesting of both species simultaneously. They showed how to classify the possibilities of the quantitative behaviour of the solutions to locate the set of initial values in which the trajectories of the solutions approach to either an asymptotic stable equilibrium or an asymptotically stable limit cycle. Recently, Dai and Tang [9] studied the following predator–prey model in which two ecological interacting species are harvested independently with constant rates:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= rx\left(1-\frac{x}{k}\right) - a\phi(x)y - \mu,\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= y(-d + ca\phi(x)) - h_1.\end{aligned} \tag{1.1}$$

They showed that system (1.1) possesses very complicated dynamics.

In this paper we consider the following set of prey–predator system:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= \alpha x\left(1-\frac{x}{k}\right) - \frac{\beta(1-m)xy}{1+a(1-m)x} - q_1E_1x,\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -\gamma y + \frac{c\beta(1-m)xy}{1+a(1-m)x} - q_2E_2y,\end{aligned} \tag{1.2}$$

where x and y denote the prey and predator population, respectively, at any time t. $\alpha > 0$ represents the intrinsic growth rate of the prey, k is the carrying capacity of the prey in the absence of predator and harvesting. The term $\beta x/(1+\alpha x)$ denotes the functional response of the predator, which is known as Holling type II response function [13]. $c > 0$ is the conversion factor denoting the number of newly born predators for each captured prey. $\gamma > 0$ is the death rate of the predator. $E_1 \geqslant 0$, $E_2 \geqslant 0$ denote the harvesting efforts for the prey and predator, respectively. q_1E_1x and q_2E_2y represent the catch of the respective species, where q_1 and q_2 represents the catchability coefficients of the prey and predator, respectively. The model incorporates a refuge protecting mx of the prey, where $m \in [0, 1)$ is constant. This leaves $(1-m)x$ of the prey available to the predator.

Mite prey–predator interactions often exhibit spatial refugia which afford the prey some degree of protection from predation and reduce the chance of extinction due to predation. Most of the theoretical and empirical prey–predator studies in ecology has focused mainly in the analysis of predator behaviour, a relatively small proportion of the ecological literature has addressed prey behaviour (see [17,11,20,8]). Refuge of prey being a natural phenomena, we have taken it into consideration.

To find models that represent stable limit cycle, an attracting self-sustained oscillation, is one of the main and primary problem in modern mathematical ecology. If a model has to describe some particular ecological system, structurally stable features (like limit cycles), which are common in real life systems, should be visible in the model. Hence, the necessity for finding conditions that guarantee the uniqueness