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Corrigendum

Corrigendum to "A candidate local minimizer of Blake and Zisserman functional" [J. Math. Pures Appl. 96 (1) (2011) 58–87]

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ABSTRACT

In a previous paper, focused on the analysis of Blake & Zisserman functional in image segmentation, we showed an Almansi-type decomposition and explicit coefficients of asymptotic expansion for bi-harmonic functions in a disk with a cut from center to boundary. The real form expansions and their subsequent applications are correct, but the auxiliary analysis of complex form expansions is imprecise. Here we wish to make precise this point.

RÉSUMÉ

Dans un article précédent, concernant la fonctionnelle de Blake et Zisserman pour la segmentation d'images, on a donné une décomposition du type d'Almansi et les coefficients explicites d'un développement asymptotique des fonctions biharmoniques dans un disque privé d'un rayon. La forme réelle du développement et les applications qui en résultent sont exactes, mais l'analyse auxiliaire du développement complexe est fausse. Cette note vise à élucider cette étape.

The main results in [7] are correct, but there is a wrong step (in the first 4 lines at page 73) in the proof of Lemma 3.5, another lack of exactness (line 8 at page 73) and two misprints in Definition 3.4 (page 71) about the notation of complex basis functions. Here we wish to amend these items, by modifying some points in Section 3 only. The other statements in Section 3 are correct and remain unchanged, except

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- substitution of the set $A^2_\varrho \setminus A^1_\varrho$ with the linear space

$$Z_\varrho = \{w(r,\vartheta) = r^2 v(r,\vartheta) : \ v \in A^1_\varrho\} \tag{1}$$

twice in the statement of Lemma 3.5 and elsewhere in the nearby comments,
- substitution of v_k and z_k in Definition 3.4 with a system of complex basis functions that exploits a more appropriate ordering and labeling:

$$\begin{aligned} v_k(r,\vartheta) &:= r^{|k|-3/2} \exp\big(i(k - 3/2 \operatorname{sign} k)\,\vartheta\big), \quad k \in \mathbb{Z} \setminus \{0\}, \\ z_k(r,\vartheta) &:= r^{|k|+1/2} \exp\big(i(k - 3/2 \operatorname{sign} k)\,\vartheta\big), \quad k \in \mathbb{Z} \setminus \{0\}. \end{aligned} \tag{2}$$

These substitutions do not affect the analysis beyond Lemma 3.5, since the results can be achieved by anticipating a slightly different proof of Theorem 3.2 (Almansi decomposition in a disk with a cut: a refinement of the classical result [1]); this new proof of Theorem 3.2, unlike the previous one, is independent of Lemma 3.8 and is contained in the new proof of Lemma 3.5. This rearrangement is necessary since the present proof of Lemma 3.5 relies on the refinement of Almansi decompositions. Here we provide a scheme with sharper statements and shorter proofs, by exploiting the present more symmetric labeling (2) of complex basis functions. The proof of Lemma 3.5 is now split in four steps (the third step provides the proof of Almansi decomposition in the disk with a cut):

I) proof of statements concerning A^1_ϱ: this item does not require substantial changes, we only add some details;
II) proof of statements concerning Z_ϱ (here replacing $A^2_\varrho \setminus A^1_\varrho$): the incorrect step is amended;
III) proof of Theorem 3.2: "if part" unchanged; "only if part" updated;
IV) proof of statements concerning A^2_ϱ: updated.

Moreover, here we make explicit the operators $\Psi : A^2_\varrho \to A^1_\varrho$ and $\Phi : A^2_\varrho \to A^1_\varrho$ associated to the Almansi decomposition in the subspace V of A^2_ϱ, which fulfills $V = \big(A^1_\varrho \cap V\big) \oplus \big(Z_\varrho \cap V\big)$, where $\oplus$ denotes the direct sum in $H^2(B_\varrho \setminus \Gamma)$. The operators Φ and Ψ allow the explicit computation of all coefficients and clarify the splitting of the expansion series studied here and in [9].

Whilst the updates consist in few pointed corrections, we write the whole statement and proof of Lemma 3.5 and the statement of Lemma 3.8 in terms of the present reordering of complex basis functions v_k and z_k to clear out all the details.

In the sequel we list the items to be substituted in [7]. We use formula labels of the kind $(1), (2), \ldots$ for formulas in the present Corrigendum and of the kind $(1^*), (2^*), \ldots$ referring to formulas in the paper [7].

Lemma 3.5. *Referring to Definition 3.4, for any $\varrho \in (0, R]$, the system*

$$\Big\{ \ v_k(r,\vartheta), \ k \in \mathbb{Z} \ \Big\} \tag{3}$$

is orthogonal in $L^2(B_\varrho)$. Moreover the system (3) is dense w.r.t. $L^2(B_\varrho)$ in the set A^1_ϱ.

The system

$$\Big\{ \ z_k(r,\vartheta), \ k \in \mathbb{Z} \ \Big\} \tag{4}$$

is orthogonal in $L^2(B_\varrho)$. Moreover the system (4) is dense w.r.t. $L^2(B_\varrho)$ in the set Z_ϱ.