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Global symmetric classical solutions of the full compressible Navier–Stokes equations with vacuum and large initial data

Huanyao Wen, Changjiang Zhu *

The Hubei Key Laboratory of Mathematical Physics, School of Mathematics and Statistics, Central China Normal University, Wuhan 430079, China

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ABSTRACT

In this paper, we get a result on global existence of classical and strong solutions of the full compressible Navier–Stokes equations in three space dimensions with spherically or cylindrically symmetric initial data which may be large. The appearance of vacuum is allowed. In particular, if the initial data is spherically symmetric, the space dimension can be taken not less than two. The analysis is based on some delicate *a priori* estimates globally in time which depend on the assumption $\kappa = O(1 + \theta^q)$ where $q > r$ (r can be zero), which relaxes the condition $q \geqslant 2 + 2r$ in [12,27,39]. This could be viewed as an extensive work of [16] where the equations hold in the sense of distributions in the set where the density is positive with initial data which is large, discontinuous, and spherically or cylindrically symmetric in three space dimension.

RÉSUMÉ

Dans cet article on obtient un résultat d'existence globale de solutions classiques et fortes des équations de Navier–Stokes complètes en trois dimensions pour des fluides incomipressibles et pour de grandes données initiales, sphériques ou cylindriques et symétriqus. L'apparition de vide est possible. En particulier si les données initiales sont sphériques ou cylindriques la dimension de l'espace peut être choisie au moins égale à deux. L'analyse utilise des estimationns *a priori* délicates globales en temps et dépendant de l'hypothèse $\kappa = O(1 + \theta^q)$ où $q > r$ (r peut être égal à zéro), ce qui affaiblit la condition $q \geqslant 2 + 2r$ donnée dans [12,27,39]. Ce résultat peut être considéré comme une extension des résultats obtenus dans les travaux [16] où les équations ont des solutions distributions dans l'ensemble où la densité est positive et les données initiales sont grandes, discontinues, sphériques ou cylindriques et symétriques dans un espace à trois dimensions.

* Corresponding author.
E-mail addresses: huanyaowen@163.com (H. Wen), cjzhu@mail.ccnu.edu.cn (C. Zhu).

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1. Introduction

The full compressible Navier–Stokes equations can be written in the sense of Eulerian coordinates in $\Omega \subset \mathbb{R}^N$ as follows:

$$\begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{u}) = 0, \\ (\rho \mathbf{u})_t + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P = \operatorname{div}(\mathcal{T}) + \rho \mathbf{f}, \\ (\rho E)_t + \operatorname{div}(\rho E \mathbf{u}) + \operatorname{div}(P\mathbf{u}) = \operatorname{div}(\mathcal{T}\mathbf{u}) + \operatorname{div}(\kappa \nabla \theta) + \rho \mathbf{u} \cdot \mathbf{f}. \end{cases} \tag{1.1}$$

Here $\mathcal{T}$ is the stress tensor given by

$$\mathcal{T} = \mu\big(\nabla \mathbf{u} + (\nabla \mathbf{u})'\big) + \lambda \operatorname{div} \mathbf{u} I_N,$$

where I_N is a $N \times N$ unit matrix; $\rho = \rho(\mathbf{x}, t)$, $\mathbf{u} = \mathbf{u}(\mathbf{x}, t) = (u_1, \ldots, u_N)(\mathbf{x}, t)$ and $\theta = \theta(\mathbf{x}, t)$ are unknown functions denoting the density, velocity and absolute temperature, respectively; $P = P(\rho, \theta)$, E, $\mathbf{f} = \mathbf{f}(\mathbf{x}, t) = (f_1, \ldots, f_N)(\mathbf{x}, t)$ and κ denote respectively pressure, total energy, external forces and coefficient of heat conduction, where $E = e + \frac{1}{2}u^2$ (e is the internal energy); μ and λ are coefficients of viscosity, satisfying the following physical restrictions:

$$\mu > 0, \quad 2\mu + N\lambda \geqslant 0;$$

P and e satisfy the second principle of thermodynamics:

$$P = \rho^2 \frac{\partial e}{\partial \rho} + \theta \frac{\partial P}{\partial \theta}. \tag{1.2}$$

(1.1) is a well-known model which describes the motion of compressible fluids. There were lots of works on the existence, uniqueness, regularity and asymptotic behavior of the solutions during the last five decades. While, because of the stronger nonlinearity in (1.1) compared with the Navier–Stokes equations for isentropic flow (no temperature equation), many known mathematical results focused on the absence of vacuum (vacuum means $\rho = 0$), refer for instance to [19,20,27,28,32,33,37] for classical solutions. More precisely, the local classical solutions to the Navier–Stokes equations with heat-conducting fluid in Hölder spaces was obtained respectively by Itaya in [19] for Cauchy problem and by Tani in [37] for IBVP with $\inf \rho_0 > 0$, where the space dimension $N = 3$. Using delicate energy methods in Sobolev spaces, Matsumura and Nishida in [32,33] showed that the global classical solutions exist provided that the initial data is small in some sense and away from vacuum in three space dimension. For large initial data in one space dimension, Kazhikhov and Shelukhi in [28] (for polytropic perfect gas with $\mu, \lambda, \kappa = \text{const}$) and Kawohl in [27] (for real gas with $\kappa = \kappa(\rho, \theta)$, $\mu, \lambda = \text{const}$) respectively got the global classical solutions to (1.1) in Lagrangian coordinates with $\inf \rho_0 > 0$. The internal energy e and the coefficient of heat conduction κ in [27] satisfy the following assumptions for $\rho \leqslant \overline{\varrho}$ and $\theta \geqslant 0$ (we translate these conditions in Eulerian coordinates)

$$\begin{cases} e(\rho, 0) \geqslant 0, \\ \nu\big(1 + \theta^r\big) \leqslant \partial_\theta e(\rho, \theta) \leqslant N(\overline{\varrho})\big(1 + \theta^r\big), \\ \kappa_0\big(1 + \theta^q\big) \leqslant \kappa(\rho, \theta) \leqslant \kappa_1\big(1 + \theta^q\big), \\ \big|\partial_\rho \kappa(\rho, \theta)\big| + \big|\partial_{\rho\rho} \kappa(\rho, \theta)\big| \leqslant \kappa_1\big(1 + \theta^q\big), \end{cases} \tag{1.3}$$

where $r \in [0, 1]$, $q \geqslant 2 + 2r$, and ν, $N(\overline{\varrho})$, κ_0 and κ_1 are positive constants. For the perfect gas (i.e., $P = R\rho\theta$, $e = C_\nu \theta$ for some constants $R > 0$ and $C_\nu > 0$) in the domain exterior to a ball in $\mathbb{R}^N$ ($N = 2$ or 3) with $\mu, \lambda, \kappa = \text{const}$, Jiang in [20] got the existence of global spherically symmetric classical large solutions in Hölder spaces.