J. Math. Pures Appl. 102 (2014) 1164–1217

Contents lists available at ScienceDirect

Journal de Mathématiques Pures et Appliquées

www.elsevier.com/locate/matpur

The formation of trapped surfaces in spherically-symmetric Einstein–Euler spacetimes with bounded variation

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ARTICLE INFO

Article history:
Received 25 December 2013
Available online 22 October 2014

MSC:
83C05
35L60
76N10

Keywords:
Einstein–Euler equations
Compressible fluid
Trapped surfaces
Spherical symmetry
Shock wave
Bounded variation

ABSTRACT

We study the evolution of a self-gravitating compressible fluid in spherical symmetry and we prove the existence of weak solutions with bounded variation for the Einstein–Euler equations of general relativity. We formulate the initial value problem in Eddington–Finkelstein coordinates and prescribe spherically symmetric data on a characteristic initial hypersurface. We introduce here a broad class of initial data which contain no trapped surfaces, and we then prove that their Cauchy development contains trapped surfaces. We therefore establish the *formation of trapped surfaces* in weak solutions to the Einstein equations. This result generalizes a theorem by Christodoulou for regular vacuum spacetimes (but without symmetry restriction). Our method of proof relies on a generalization of the "random choice" method for nonlinear hyperbolic systems and on a detailed analysis of the nonlinear coupling between the Einstein equations and the relativistic Euler equations in spherical symmetry.

RÉSUMÉ

On étudie l'évolution d'un fluide compressible auto-gravitant en symétrie radiale et démontre un résultat d'existence de solutions faibles à variation bornée pour les équations d'Einstein–Euler de la relativité générale. On formule le problème de Cauchy en coordonnées d'Eddington–Finkelstein et on prescrit des données à symétrie radiale sur une hypersurface initiale caractéristique. On introduit ici une classe de données initiales qui ne contiennent pas de surfaces piégées, et on démontre alors que leur développement de Cauchy contient des surfaces piégées. On établit ainsi un résultat de *formation de surfaces piégées* dans les solutions faibles des équations d'Einstein. Ce résultat généralise un théorème de Christodoulou pour les espaces–temps réguliers sans matière (mais sans restriction de symétrie). Notre méthode de démonstration s'appuie sur une généralisation de la méthode « random choice » pour les systèmes hyperboliques non linéaires et sur une analyse fine

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http://dx.doi.org/10.1016/j.matpur.2014.10.003

du couplage non linéaire entre les équations d'Einstein et les équations d'Euler relativistes en symétrie radiale.

1. Introduction

We are interested in the problem of the gravitational collapse of compressible matter under the assumption of spherical symmetry. When the matter evolves under its self-induced gravitational field, two distinct behaviors can be observed: a dispersion of the matter in future timelike directions, or a collapse of the matter and the formation of a trapped surface and, under certain conditions, a black hole [14,22,29]. The collapse problem in spherical symmetry was extensively investigated by Christodoulou and followers in the past twenty years, under the assumption that the matter is represented by a scalar field [4,5] or is driven by a kinetic equation like Vlasov equation; cf. Andreasson [1], Andreasson and Rein [2], and Rendall [24,25] and the references cited therein. Furthermore, the problem of the generic formation of trapped surfaces in vacuum spacetimes without symmetry was solved by Christodoulou in the pioneering work [6].

In recent years, the second author together with collaborators [3,12,16,18,20,21] has initiated the mathematical study of self-gravitating compressible fluids and constructed classes of spacetimes with weak regularity whose curvature is defined in the sense of distributions [17]. Global existence results have been established for several classes of solutions to the Einstein equations with symmetry. LeFloch and Stewart [21] proposed a mathematical theory of the characteristic initial value problem for plane-symmetric spacetimes with weak regularity, while LeFloch and Rendall [18] and Grubic and LeFloch [12] constructed a global foliation for the larger class of weakly regular spacetimes with Gowdy symmetry. Furthermore, LeFloch and Smulevici [19] developed the theory of weakly regular, vacuum spacetimes with T^2 symmetry.

The present paper is motivated by Christodoulou's work [6] on trapped surface formation and, by building upon the mathematical technique [3,16,18,21], we are able to construct a large class of spherically-symmetric Einstein–Euler spacetimes which have bounded variation and exhibit trapped surface formation. We thus consider matter spacetimes $(\mathcal{M}, g)$ (with bounded variation) satisfying the Einstein equations

$$G^{\alpha\beta} = 8\pi T^{\alpha\beta} \tag{1.1}$$

understood in the distributional sense (see Section 3, below), when the geometry described by the Einstein tensor $G^{\alpha\beta}$ is coupled to the matter content governed by the energy–momentum tensor

$$T^{\alpha\beta} = (\mu + p)u^\alpha u^\beta + pg^{\alpha\beta}. \tag{1.2}$$

Here, all Greek indices take values $0, \dots, 3$ and implicit summation over repeated indices is used. According to the Bianchi identities satisfied by the geometry, (1.1)–(1.2) imply the Euler equations

$$\nabla_\alpha T^{\alpha\beta} = 0. \tag{1.3}$$

In (1.2), μ denotes the mass–energy density of the fluid and u^α its velocity vector, which is normalized to be of unit norm $u^\alpha u_\alpha = -1$, while the pressure function $p = p(\mu)$ is assumed to depend linearly on μ, that is,

$$p = k^2\mu. \tag{1.4}$$

The constant $k \in (0, 1)$ represents the speed of sound, while the light speed is normalized to unit.

In the present paper, we thus investigate the class of spherically symmetric spacetimes governed by the Einstein–Euler equations (1.1)–(1.3), and after formulating the initial value problem with data posed on a