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Stability of an inverse problem for the discrete wave equation and convergence results [☆]

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A B S T R A C T

Using uniform global Carleman estimates for semi-discrete elliptic and hyperbolic equations, we study Lipschitz and logarithmic stability for the inverse problem of recovering a potential in a semi-discrete wave equation, discretized by finite differences in a 2-d uniform mesh, from boundary or internal measurements. The discrete stability results, when compared with their continuous counterparts, include new terms depending on the discretization parameter h. From these stability results, we design a numerical method to compute convergent approximations of the continuous potential.

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R É S U M É

A partir d'inégalités de Carleman pour des équations aux dérivées partielles dicrétisées elliptiques et hyperboliques, on étudie la stabilité Lipschitz et logarithmique du problème inverse de détermination du potentiel dans une équation des ondes semidiscrétisée, par un schéma aux différences finies sur un maillage 2-d uniforme, à partir de mesures internes ou frontières. Quand ils sont comparés avec leur contrepartie continue, les résultats de stabilité dans le cadre discret contiennent de nouveaux termes dépendants du pas h du maillage utilisé. C'est à partir de ces résultats qu'on donne une méthode numérique de calcul d'approximations convergentes du potentiel continu.

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1. Introduction

The goal of this article is to study the convergence of an inverse problem for the wave equation, which consists in recovering a potential through the knowledge of the flux of the solution on a part of the boundary. This article follows the previous work [2] on that precise topic in the 1-d case.

1.1. The continuous inverse problem

Setting. We will first present the main features of the continuous inverse problem we consider in this article. Let Ω be a smooth bounded domain of $\mathbb{R}^d$, and for $T > 0$, consider the wave equation:

$$\begin{cases} \partial_{tt} y - \Delta y + q y = f, & \text{in } (0,T) \times \Omega, \\ y = f_\partial, & \text{on } (0,T) \times \partial\Omega, \\ y(0,\cdot) = y^0, \quad \partial_t y(0,\cdot) = y^1, & \text{in } \Omega. \end{cases} \tag{1.1}$$

Here, $y = y(t,x)$ is the amplitude of the waves, (y^0, y^1) is the initial datum, $q = q(x)$ is a potential, f is a distributed source term and f_∂ is a boundary source term.

In the following, we explicitly write down the dependence of the function y solution of (1.1) in terms of q by denoting it $y[q]$ and similarly for the other quantities depending on q.

We assume that the initial datum (y^0, y^1) and the source terms f and f_∂ are known. We also assume the additional knowledge of the flux

$$\mathscr{M}[q] = \partial_\nu y[q] \quad \text{on } (0,T) \times \Gamma_0, \tag{1.2}$$

where Γ_0 is a non-empty open subset of the boundary $\partial\Omega$ and ν is the unit outward normal vector on $\partial\Omega$. Note that for this map to be well-defined, we need to give a precise functional setting: for instance, we may assume $(y^0, y^1) \in H^1(\Omega) \times L^2(\Omega)$, $f \in L^1((0,T); L^2(\Omega))$, $f_\partial \in H^1((0,T) \times \partial\Omega)$ and $y^0|_{\partial\Omega} = f_\partial(t=0)$ so that $\mathscr{M}$ is well-defined for all $q \in L^\infty(\Omega)$ and takes value in $L^2((0,T) \times \partial\Omega)$, see e.g. [27].

This article is about the recovering the potential q from $\mathscr{M}[q]$. As usual when considering inverse problems, this topic can be decomposed into the following questions:

- Uniqueness: Does the measurement $\mathscr{M}[q]$ uniquely determine the potential q?
- Stability: Given two measurements $\mathscr{M}[q^a]$ and $\mathscr{M}[q^b]$ which are close, are the corresponding potentials q^a and q^b close?
- Reconstruction: Given a measurement $\mathscr{M}[q]$, can we design an algorithm to recover the potential q?

Concerning the precise inverse problem we are interested in, the uniqueness result is due to [11] and we shall focus on the stability properties of the inverse problem (1.1). The question of stability has attracted a lot of attention and is usually based on Carleman estimates. There are mainly two types of results: Lipschitz stability results, see [25,31,32,38,22,1,23,3,35], provided the observation is done on a sufficiently large part of the boundary and the time is large enough, or logarithmic stability results [4,6] when the observation set does not satisfy any geometric requirement. We also mention the works [5,12] for logarithmic stability of inverse problems for other related equations.

Below we present more precisely these two type of results, since our main goal will be to discuss discrete counterparts in these two cases.

Lipschitz stability results under the Gamma-conditions. Getting Lipschitz stability results for the continuous inverse problem usually requires the following assumptions, originally due to [18]. We say that the triplet (Ω, Γ, T) satisfy the Gamma-conditions (see [29]) if

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