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The Berglund–Hübsch–Chiodo–Ruan mirror symmetry for K3 surfaces

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ABSTRACT

We prove that the mirror symmetry of Berglund, Hübsch, Chiodo and Ruan, applied to K3 surfaces with a non-symplectic involution, coincides with the lattice mirror symmetry.

RÉSUMÉ

On montre que la symétrie miroir de Berglund, Hübsch, Chiodo et Ruan, appliquée aux surfaces K3 munies d'une involution non-symplectique, coïncide avec la symétrie miroir réticulaire.

1. Introduction

The most famous example of mirror symmetry between Calabi–Yau varieties was given in 1991 by the physicists Candelas, de la Ossa, Green and Parkes [1], where they described the mirror family of a one parameter family of smooth quintic threefolds in $\mathbb{P}^4$. The mirror family is a desingularization of a quotient of the family by a finite group acting symplectically on it. Since then a big amount of work has been done, and mirror symmetry has found its expression in many (mathematical) ways e.g. through toric geometry or Landau–Ginzburg theory. In 1992, Berglund and Hübsch [2] described a special construction of mirror pairs of Calabi–Yau manifolds given as finite quotients of certain hypersurfaces in weighted projective spaces extending the construction of [1]. Later, Chiodo and Ruan in [3], using results of Krawitz in [4] proved that the transposition rule of Berglund and Hübsch provides pairs of Calabi–Yau manifolds whose Hodge diamonds

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have the symmetry required in mirror symmetry. In this paper we apply the transposition rule to certain K3 surfaces carrying a non-symplectic involution and we relate this to a mirror construction between families of lattice polarized K3 surfaces (we call it *lattice mirror symmetry*) due to Dolgachev and Nikulin [5–7], Voisin [8] and Borcea [9]. Our main Theorem 1.1 states that the transposition rule by Berglund and Hübsch, in this case, provides pairs of K3 surfaces which belong to lattice mirror families. The results of [7] and [8, Lemma 2.5 and §2.6] are in particular fundamental for our theorem (see Subsection 4.2).

Let W denote a Delsarte type polynomial, i.e. a polynomial having as many monomials as variables (this will be called "potential" in the sequel, following the terminology of physicists). Assume that the matrix A of exponents of W is invertible over $\mathbb{Q}$, that $\{W = 0\}$ has an isolated singularity at the origin and that it defines a well-formed hypersurface in some normalized weighted projective space. We denote by $\mathrm{Aut}(W)$ the group of diagonal symmetries of W, by $\mathrm{SL}(W)$ the group of diagonal symmetries of determinant one, and by J_W the monodromy group of the affine Milnor fiber associated to W. Let W^T denote the *transposed* potential defined by the matrix A^T. For any subgroup $G \subset \mathrm{Aut}(W)$, we denote by G^T the *transposed* group of automorphisms of the potential W^T, this can be defined as in [10, Section 4] as the kernel of the dual morphism between the dual groups $\mathrm{Aut}(W)^* \to G^*$ (see Section 3 for many equivalent descriptions of G^T). The main result of this paper is the following.

Theorem 1.1. *Let W be a K3 surface defined by a non-degenerate and invertible potential of the form*:

$$x^2 = f(y, z, w)$$

in some weighted projective space. Let G_W be a subgroup of $\mathrm{Aut}(W)$ such that $J_W \subset G_W \subset \mathrm{SL}(W)$. Put $\widetilde{G_W} := G_W/J_W$ and $\widetilde{G_W^T} := G_W^T/J_{W^T}$. Then the Berglund–Hübsch–Chiodo–Ruan mirror orbifolds $[W/\widetilde{G_W}]$ and $[W^T/\widetilde{G_W^T}]$ belong to lattice mirror families.

The Berglund–Hübsch–Chiodo–Ruan (BHCR for short) mirror symmetry applies to Calabi–Yau varieties in weighted projective spaces which are not necessarily Gorenstein. As remarked by Chiodo and Ruan in [3, Section 1], this is the main difference with Batyrev mirror symmetry [11]. Most of our K3 surfaces are not contained in a Gorenstein weighted projective space.

The paper is organized as follows. In Section 2 we give some preliminary results about hypersurfaces in weighted projective spaces, potentials and the Berglund–Hübsch construction. In Section 3 we describe the group $\mathrm{Aut}(W)$ of diagonal automorphisms of a potential and we make a bridge between the different descriptions of the transposed group G^T of a subgroup G of $\mathrm{Aut}(W)$. Most of the results of the section are already contained in (or follow directly from results contained in) [4] and [12]. We give easier proofs using some simple (linear) algebra. Section 4 contains preliminary facts about non-symplectic involutions on K3 surfaces and introduces the lattice mirror construction. Section 5 deals with K3 surfaces defined by a potential as in the statement of Theorem 1.1: we study their singularities and we determine the basic invariants of the non-symplectic involution $x \mapsto -x$. In Section 6 we give the proof of Theorem 1.1.

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2. The Berglund–Hübsch–Chiodo–Ruan construction

2.1. Hypersurfaces in weighted projective spaces

We start by recalling some basic facts about hypersurfaces in weighted projective spaces, see for example [13]. Let $x_1, \dots, x_n$ be affine coordinates on $\mathbb{C}^n$, $n \geqslant 3$, and let $(w_1, \dots, w_n)$ be a sequence of positive weights. The group $\mathbb{C}^*$ acts on $\mathbb{C}^n$ by