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Stability of non-constant equilibrium solutions for Euler–Maxwell equations

Yue-Jun Peng [a,b]

[a] *Clermont Université, Université Blaise Pascal, 63000 Clermont-Ferrand, France*
[b] *Laboratoire de Mathématiques, CNRS-UMR 6620, Complexe Scientifique Les Cézeaux, BP 80026, 63171 Aubière cedex, France*

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ABSTRACT

We consider a periodic problem for compressible Euler–Maxwell equations arising in the modeling of magnetized plasmas. The equations are quasilinear hyperbolic and partially dissipative. It is proved that smooth solutions exist globally in time and converge toward non-constant equilibrium states as the time goes to infinity. This result is obtained for initial data close to the equilibrium states with zero velocity. The proof is based on an induction argument on the order of the derivatives of solutions in energy and time dissipation estimates. We also show the global stability with exponential decay in time of solutions near the equilibrium states for compressible Euler–Poisson equations.

RÉSUMÉ

On considère un problème périodique pour les équations d'Euler–Maxwell compressibles modélisant des plasmas magnétisés. Ces équations sont hyperboliques quasilinéaires et partiellement dissipatives. On démontre que les solutions régulières existent globalement en temps et convergent vers des états d'équilibre non constants lorsque le temps tend vers l'infini. Ce résultat est obtenu pour les données initiales proches des états d'équilibre avec vitesse nulle. La démonstration utilise un argument de récurrence sur l'ordre des dérivées des solutions dans des estimations d'énergie et de dissipation en temps. On montre aussi la stabilité globale avec décroissance exponentielle en temps des solutions proches des états d'équilibre pour les équations d'Euler–Poisson compressibles.

1. Introduction

This paper is concerned with the stability of non-constant equilibrium solutions for Euler–Maxwell equations arising in the modeling of magnetized plasmas. We suppose that the plasmas are composed of

E-mail address: peng@math.univ-bpclermont.fr.

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two types of particles, electrons and ions, with fixed density of ions. This leads to a one-fluid compressible Euler–Maxwell system for electrons. Let n and u be the density and the velocity vector of the fluid for electrons, E and B be, respectively, the electric field and the magnetic field of a magnetized plasma. The fields E and B are coupled with (n, u) through the Maxwell equations and act on them via the Lorentz force. These variables are functions of the time $t > 0$ and the position $x \in \mathbb{R}^3$. In this paper, we consider the periodic problem in a three-dimensional torus $\mathbb{T}^3 \stackrel{\text{def}}{=} (\mathbb{R}/\mathbb{Z})^3$.

The one-fluid compressible Euler–Maxwell system reads (see [2,4,27]):

$$\begin{cases} \partial_t n + \operatorname{div}(nu) = 0, \\ \partial_t(nu) + \operatorname{div}(nu \otimes u) + \nabla p(n) = -n(E + u \times B) - nu, \\ \partial_t E - \nabla \times B = nu, \quad \operatorname{div} E = b - n, \\ \partial_t B + \nabla \times E = 0, \quad \operatorname{div} B = 0, \end{cases} \tag{1.1}$$

for $(t, x) \in (0, +\infty) \times \mathbb{T}^3$. Here the given functions b and p stand for the ion density and the pressure, respectively. As usual, we suppose that p is a smooth and strictly increasing function on $(0, +\infty)$. This includes the usual state equation of γ-law: $p(n) = n^\gamma$, with $\gamma \geqslant 1$. The function b depends only on x, and it is compatible with system (1.1). Indeed, we have

$$\partial_t n = -\operatorname{div}(nu) = -\partial_t(\operatorname{div} E) = \partial_t n - \partial_t b,$$

which implies that $\partial_t b = 0$. Since we consider periodic smooth solutions, b is supposed to be smooth and periodic. Moreover, we suppose that $b \geqslant \text{const.} > 0$ in $\mathbb{T}^3$. System (1.1) is complemented by initial conditions:

$$t = 0: \quad (n, u, E, B) = \left(n^0, u^0, E^0, B^0\right), \quad \text{in } \mathbb{T}^3. \tag{1.2}$$

In (1.1), all physical parameters are set equal to unity. This is not an essential restriction in the study of global existence of smooth solutions near equilibrium states. Otherwise, the smallness conditions on the initial data in the main results would depend on the parameters. We refer to [24–26,29,30] and references therein for descriptions and asymptotic analysis for (1.1) with various physical parameters. In the momentum equations of (1.1), the quantity $n(E+u\times B)$ stands for the Lorentz force and $-nu$ is the relaxation damping. For smooth solutions with $n > 0$, these equations are equivalent to

$$\partial_t u + (u \cdot \nabla)u + \nabla h(n) = -(E + u \times B) - u, \tag{1.3}$$

where $\cdot$ denotes the inner product of $\mathbb{R}^3$ and h is the enthalpy function defined by

$$h(n) = \int_1^n \frac{p'(\xi)}{\xi}\, d\xi.$$

Since p is strictly increasing on $(0, +\infty)$, so is h. It is well known that the constraint equations

$$\operatorname{div} E = b - n, \qquad \operatorname{div} B = 0 \tag{1.4}$$

are time invariant. Therefore, we have to make the following assumption:

$$\operatorname{div} E^0 = b - n^0, \qquad \operatorname{div} B^0 = 0. \tag{1.5}$$