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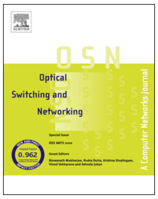

Cost-efficient algebraic connectivity optimisation of backbone networks

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ABSTRACT

Backbone networks are prone to failures due to targeted attacks or large-scale disasters. Network resilience can be improved by adding new links to increase network connectivity and robustness. However, random link additions without an optimisation objective function can have insignificant connectivity improvement. In this paper, we develop a heuristic algorithm that optimises a network by adding links to achieve a higher network resilience by maximising algebraic connectivity and decreasing total cost via selecting cost-efficient links. We apply our algorithm to five different backbone topologies and measure algebraic connectivity improvement and the cost incurred while adding new links. For evaluation, we apply three centrality node attacks to the non- and optimised networks and show the network flow robustness while nodes are removed. Our results show that optimised graphs with higher algebraic connectivity values are mostly more resilient to centrality-based node attacks.

1. Introduction and motivation

Networks in general, and communication networks in particular, are prone to a variety of challenges and attacks that can have costly consequences. However, network connectivity can be improved with careful planning and optimisation, and the impact of such challenges can be reduced. The design and optimisation of cost-efficient networks that are resilient against challenges and attacks has been studied by many researchers over the past few decades, but the resilient network design problem is NP-hard.

In this paper, we approach resilient network design from a graph theoretic perspective. We develop a *heuristic algorithm* that improves the connectivity of a graph in terms of the *algebraic connectivity* metric by adding links. Algebraic connectivity $a(G)$ is defined as the second smallest eigenvalue of the Laplacian matrix [1] and it is widely used for topological optimisations [2–4]. A secondary objective of our algorithm is to select the links that improve the algebraic connectivity of the graph in the least costly fashion in which we capture the cost of network as the total link length. Furthermore, we parameterise our optimisation algorithm such that connectivity and cost are weighted depending on a cost-effect parameter γ.

The heuristic to increase algebraic connectivity in a graph is based on adding links to the nodes that have least incident links (i.e. minimal degree nodes) [2,4]. Our parameterised heuristic algorithm identifies and selects the links that increase the algebraic connectivity of a graph

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depending on the available budget. Moreover, the search of the best links is computationally less expensive in our algorithm compared to an exhaustive search. We use five commercial service provider physical networks (AT&T, Level 3, Sprint, Internet2, and CORONET) to evaluate our algorithm. Our algorithm provides the cost-efficient new links to improve a network's resilience measured by the algebraic connectivity metric.

The rest of the paper is organised as follows: we present a brief background on network design and optimisation in Section 2. The assumptions, objective functions, and our heuristic algorithm are presented in Section 3. The dataset for the communication networks as well as optimisation evaluation of these topologies using our algorithm is presented in Section 4. The robustness evaluation of the non- and optimised graphs is presented in Section 5. Finally, we summarise our findings as well as propose future work in Section 6.

2. Background and related work

Network design and optimisation have been studied in past few decades [5–14] and many problems in this field are considered to be NP-hard [15–18]. Several monographs provide in-depth coverage of the topic [19–22]. The design process includes constructing the network from the ground up including placement of nodes [8,9] and providing connectivity among nodes to enable services. The optimisation process includes improvement of the network for one or multiple objectives. Network optimisation can be accomplished by means of rewiring while keeping the number of edges constant [4] or by means of adding new links to improve the connectivity of graphs [2]. Moreover, the design process is different for backbone and access networks, since the topological structure of these networks fundamentally differs [8–10].

Network design and optimisation objectives are cost, capacity, reliability, and performance [7–9]. Network cost is incurred by the number of nodes required, capacity of nodes required, and number of links. Previously, we provided a network cost model to add a link between node i and node j as

$$C_{i,j} = f + v \times d_{i,j} \tag{1}$$

where f is the fixed cost associated with the link (including termination), v is the variable cost per unit distance for the link, and $d_{i,j}$ is the length of the link [23,24]. Moreover, if we assume that the fibre length dominates *wide-area* network cost and ignore the fixed cost associated with each link, the network cost can be written as

$$C = \sum_i l_i \tag{2}$$

where l_i is the length of the ith link [25,26].

Topological connectivity is another objective that can be measured by means of many graph metrics such as average degree, betweenness, closeness, and graph diversity [4–6,27–29]. In this paper, we measure the connectivity of a graph in terms of algebraic connectivity metric. Algebraic connectivity $a(G)$ is defined as the second smallest eigenvalue of the Laplacian matrix [1]. The Laplacian matrix of G is $L(G) = D(G) - A(G)$ where $D(G)$ is the diagonal matrix of node degrees, $d_{ii} = \deg(v_i)$, and $A(G)$ is the symmetric adjacency matrix with no self-loops. The algebraic connectivity of a complete graph (i.e. full mesh) is n where n is the number of nodes, and it is 0 for a disconnected graph with more than one component.

Topology design using algebraic connectivity has been studied by several researchers [2–4]. It has been shown that algebraic connectivity is more informative and accurate than average node degree when characterising network resilience [3]. Moreover, we have shown algebraic connectivity [26,30] and diversity [29] are predictive of flow robustness of graphs. Three synthetically generated topologies (i.e. Watts–Strogatz, Gilbert, Barabási-Albert) have been optimised using edge rewiring in which the objective is to increase the algebraic connectivity [4]. It was shown that algebraic connectivity increases the most if edges are rewired between weakly connected nodes. Another study optimised synthetically generated Erdős–Rényi and Barabási–Albert graphs in terms of adding links to the existing topology [2]. It was concluded that adding links between a low degree node and a random node is computationally less expensive than an exhaustive search. In this paper, we present an algorithm for topological optimisation in terms of adding links, which maximises algebraic connectivity and aims to choose links so that the cost is minimal among given choices.

3. Topology optimisation algorithm

In this section, we describe our algorithm that optimises connectivity and cost of a topology. Our heuristic algorithm is implemented using Python. Furthermore, we assume that node locations are given for a graph to apply optimisation algorithm, as would be the case for a deployed service provider.

3.1. Objectives

The objective of this algorithm is to identify the best links to be added to improve the connectivity of the graph. In this paper, we use algebraic connectivity as a measure of connectivity, but we note that any graph connectivity property, such as average node degree, clustering coefficient, or diversity can be used with our heuristic. For example, the clustering coefficient can be used to replace the algebraic connectivity or both the clustering coefficient and the algebraic connectivity can be used with a tuning parameter to control their effect in selecting the links.

3.2. Algorithm

The topology optimisation algorithm has three inputs: an input graph G_i, a number of required links L_r, and a cost-effect parameter γ. The input graph G_i has a number of nodes n_i with a number of links l_i. The number of required links L_r is the number of links that should be added to the graph. The cost-effect parameter γ is a tuning parameter between cost and algebraic connectivity. When $\gamma = 0$, the cost term of the rank function is neglected since it is zeroed. As a result, the algorithm selects the link that