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Monte Carlo methods in diameter-constrained reliability

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ABSTRACT

A classical requirement in the design of communication networks is that all entities must be connected. In a network where links may fail, the connectedness probability is called all-terminal reliability. The model is suitable for FTTH services, where link failures are unpredictable. In real scenarios, terminals must be connected by a limited number of hops. Therefore, we study the Diameter-Constrained Reliability (DCR). We are given a simple graph $G=(V,E)$, a subset $K \subseteq V$ of terminals, a diameter d and independent failure probabilities $q=1-p$ for each link. The goal is to find the probability $R^d_{K,G}$ that all terminals remain connected by paths composed by d hops or less. The general DCR computation is $\mathcal{NP}$−Hard, and the target probability is a polynomial in p.

In this paper we study the DCR metric. It connects reliability with quality, and should be considered in the design of the physical layer in FTTH services together with connectivity requirements. We include a full discussion of the computational complexity of the DCR as a function of the number of terminals $k=|K|$ and diameter d. Then, we find efficient DCR computation for Monma graphs, an outstanding family of topologies from robust network design. The computation suggests corollaries that enrich the subset of instances that accept efficient DCR computation.

Inspired in its $\mathcal{NP}$−Hardness, we introduce two approximation algorithms in order to find the DCR in general. The first one estimates the target polynomial counting special subgraphs. The second finds pointwise estimations of the polynomial using conditioned-Monte Carlo, and applies Newton's interpolation followed by a rounding stage of the coefficients.

The performance of both methods is discussed on the lights of Complete, Harary and Monma graphs. In order to study scalability, we analytically find the diameter-constrained reliability of a series–parallel graph with 44 nodes and 72 links. The results suggest that our counting implementation outperforms the interpolation technique, and is scalable as well. Open problems and trends for future work are included in the conclusions.

1. Motivation

A hot-topic in the fiber optic field is to design an IP/MPLS network over a DWDM physical layer, trying to integrate traffic and capacity constraints. Usually, the operator needs to fulfill several requirements, as resilience against single-point of failures (by means of 2-node connectivity in order not to have service disruption), the delicate mapping from logical into physical layer (trying to save bandwidth access and trading high-connectivity with loss probability), and the physical construction properly [28,29].

The design of the physical layer must overcome several challenges. Among them, we recognize four dimensions in

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the design of the physical layer: *availability*, *survivability*, *quality* and *reliability*. Availability is the capacity of the network to maintain the service everywhere and every time. Since optical fibers are rich in bandwidth access, a spanning network seems enough to keep the system operational. Survivability is the capacity of a network to survive under several failures. The general approach is to consider redundancy: trees are minimum connected structures, but if a link fails the system goes down. Therefore, the original tree-like idea is not enough. Indeed, it is widely accepted that the physical layer should be at least 2-node connected (i.e. the system keeps alive even under an arbitrary link or node failure).

Another dimensions that are rarely found in the literature of fiber optics are quality and reliability. Quality usually depends on the application under study. The most demanding one from a network viewpoint is video streaming over the Internet. There are objective, subjective and more recently, pseudo-subjective metrics for quality assessment (namely, Pseudo Subjective Quality Assessment, or PSQA [37]). Experimental works confirm that the most shocking factors in video quality are start-up latency and playback continuity [32]. The reader can find playback-delivery trade-offs in [35,33]. Playback continuity can be tackled either by a full-cooperative environment or powerful Content Delivery Networks (CDN), where servers concentrate and distribute all the traffic load. Start-up latency can be reduced by several means, remarkably, with multiple seeds and hop-constraints. Observe that survivability considers a deterministic nature of the network topology [30]. Reliability is another dimension, where links (or nodes) can fail with certain probabilities, and the goal is to keep a specific network property. In historical networks (PSTN for instance) the required structure was just connectedness, and the focus was mainly availability [23].

A foundational work in robust network design is owned by Monma et. al [24]. The authors showed that optimal topologies in the minimum-cost two-node-connected network design must either be Hamiltonian or present a special graph topology as an induced subgraph. This topology is sketched in Fig. 1. It has exactly two vertices with degree 3, and three independent paths connecting them. They are called Monma graphs for the first time in [8].

In this paper we connect reliability and quality aspects of network design. We assume a scenario where bandwidth access is not a problem, provided all nodes are reachable with few hops, inspired in latency-driven applications. With no a priori information, a common rule of thumb is to consider the failure event of a fiber as a stochastic one, where all links may fail independently, with a certain probability $q = 1-p$ (the number p denotes the probability of operation). Inspired in a low-cost design and with a reverse-engineering approach, we want to find the connectedness probability of the physical topology. Since there are outstanding nodes (i.e. terminals), and guided by latency constrained applications (VoIP, live video-streaming, peer-to-peer networks, flooding-based systems), we additionally require that all terminals remain connected with paths composed by d hops, or less. This probability, called *diameter-constrained reliability*, is a metric that integrates both reliability and quality (inspired in delay-sensitive applications), where the terminal set $K \subseteq V$ is normally the *core* of the network.

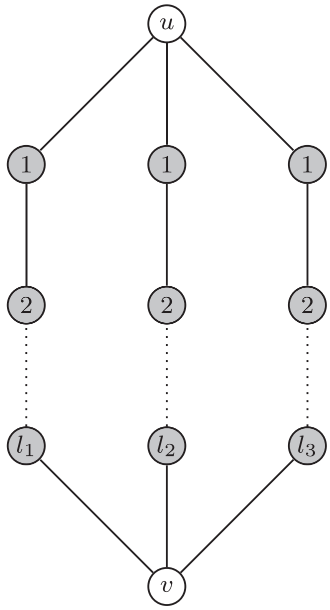

Fig. 1. Monma's graph structure.

The classical reliability (CLR) metric is connectedness probability, and has a rich history and applicability, ranging from transportation systems to grid computing, electrical networks and telecommunications in a wide sense. A natural generalization is "d-connectedness", this is, to require the existence of paths not longer than d, for each pair of terminals. This problem, called *diameter-constrained reliability* and denoted DCR for short, is largely inspired in real-life applications [11,13,14].

Our goal is to efficiently find the diameter-constrained reliability of an arbitrary network, with particular focus on robust networks. This document is organized in the following manner. Section 2 formally presents the DCR, and sketches the surface of non-exact approaches to find the target probabilities. Section 3 presents a full analysis of computational complexity of the DCR, in terms of the diameter d and number of terminals $k = |K|$. In Section 4, we develop exact computations of the DCR for special topologies. In particular, the DCR for Monma graphs can be found in polynomial time with respect to the order of the graph. Since the DCR is an $\mathcal{NP}$−Hard problem, approximation algorithms represent an essential tool. The main contributions are summarized in Section 5. There, the traditional design of Monte-Carlo methods is presented in a macroscopic viewpoint. Then, two novel Monte-Carlo based methods are designed in order to evaluate the DCR for an arbitrary network. In Section 6, the effectiveness of both approaches is analyzed on the lights of Complete, Harary and Monma graphs, and a series–parallel graph with 44 nodes and 72 links. The bridge between classical interpolation and network reliability is discussed in