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The basins of attraction of Murakami's fifth order family of methods

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ABSTRACT

In this paper we analyze Murakami's family of fifth order methods for the solution of nonlinear equations. We show how to find the best performer by using a measure of closeness of the extraneous fixed points to the imaginary axis. We demonstrate the performance of these members as compared to the two members originally suggested by Murakami. We found several members for which the extraneous fixed points are on the imaginary axis, only one of these has 6 such points (compared to 8 for the other members). We show that this member is the best performer.

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1. Introduction

There is a vast literature on the solution of nonlinear equations, see for example Ostrowski [19], Traub [23], Neta [16] and Petković et al. [20]. In this paper we consider a fifth-order family of methods and show how to choose the best parameters. We will compare the performance of the two originally suggested members to two new ones by using the idea of basin of attraction and analyzing the extraneous fixed points.

Murakami [15] has developed a fifth order family of methods

$$x_{n+1} = x_n - a_1 u_n - a_2 w_2(x_n) - a_3 w_3(x_n) - \psi(x_n), \tag{1}$$

where

$$\begin{aligned} u_n &= \frac{f(x_n)}{f'(x_n)}, \\ w_2(x_n) &= \frac{f(x_n)}{f'(x_n - u_n)}, \end{aligned} \tag{2}$$

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$$w_3(x_n) = \frac{f(x_n)}{f'(x_n+\beta u_n+\gamma w_2(x_n))},$$

$$\psi(x_n) = \frac{f(x_n)}{b_1 f'(x_n)+b_2 f'(x_n-u_n)}.$$

This family is of order five when we take

$$a_1 = \frac{1}{6}\left(1+\frac{4\gamma+1}{\theta}\right), \qquad a_2 = \frac{1}{\theta-1}\left(\frac{1}{6}\theta-\frac{2}{3}\gamma-\frac{1}{3}\right), \qquad a_3 = \frac{2}{3},$$
$$b_1 = -\frac{6\theta(\theta-1)^2}{4\gamma+1}, \qquad b_2 = \frac{6\theta^2(\theta-1)}{4\gamma+1}, \qquad \beta = -\gamma-\frac{1}{2}, \tag{3}$$

and

$$\theta = \frac{16\gamma+5}{4(4\gamma+1)}. \tag{4}$$

Murakami suggested the following two possibilities:

$$\gamma = 0, \qquad a_1 = 0.3, \qquad a_2 = -0.5, \qquad a_3 = \frac{2}{3},$$
$$b_1 = -\frac{15}{32}, \qquad b_2 = \frac{75}{32}, \qquad \beta = -\frac{1}{2} \tag{5}$$

and

$$\gamma = -0.5, \qquad a_1 = -\frac{1}{18}, \qquad a_2 = -\frac{1}{2}, \qquad a_3 = \frac{2}{3},$$
$$b_1 = \frac{9}{32}, \qquad b_2 = \frac{27}{32}, \qquad \beta = 0. \tag{6}$$

The idea there probably to choose one of the parameters to be zero, i.e. either $\gamma = 0$ or $\beta = 0$. As it turns out these parameters are not far from the best.

In this paper, we find the best possible value of the parameter γ. We will use two criteria we have developed in previous work [7] based on the location of the extraneous fixed points. In the next section, we discuss the extraneous fixed points. In section 3 we will discuss the two criteria and give the best parameter based on these criteria. In section 4 we describe the basins of attraction for the best members of the family for 7 different examples. We close with conclusions.

2. Extraneous fixed points

For the Murakami family $z_{n+1} = M_f(z_n)$, where

$$M_f(z) = z - a_1 u_f(z) - a_2 w_{2,f}(z) - a_3 w_{3,f}(z) - \psi_f(z),$$

$$u_f(z) = \frac{f(z)}{f'(z)},$$

$$w_{2,f}(z) = \frac{f(z)}{f'(z-u_f(z))}, \tag{7}$$

$$w_{3,f}(z) = \frac{f(z)}{f'(z+\beta u_f(z)+\gamma w_{2,f}(z))},$$

$$\psi_f(z) = \frac{f(z)}{b_1 f'(z)+b_2 f'(z-u_f(z))},$$

we explore its conjugacy on quadratic polynomials. We begin with a preliminary result.

Lemma 1. *Let $f(z)$ be an analytic function on the Riemann sphere, and let $T(z) = \alpha z+\beta$, $\alpha \neq 0$, be an affine map. If $g(z) = (f \circ T)(z)$, then we have*