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A radial basis function based implicit–explicit method for option pricing under jump-diffusion models

Mohan K. Kadalbajoo, Alpesh Kumar*, Lok Pati Tripathi

Department of Mathematics and Statistics, Indian Institute of Technology Kanpur, Kanpur 208016, India

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A B S T R A C T

In this article, we present a radial basis function based implicit explicit numerical method to solve the partial integro-differential equation which describes the nature of the option price under jump diffusion model. The governing equation is time semi discrtized by using the implicit–explicit backward difference method of order two (IMEX-BDF2) followed by radial basis function based finite difference (RBF-FD) method. The numerical scheme derived for European option is extended for American option by using operator splitting method. Numerical results for put and call option under Merton and Kou models are given to illustrate the efficiency and accuracy of the present method. The stability of time semi discretized scheme is also proved.

1. Introduction

There is evidence to suggest that the Black Scholes model for stock price behavior does not always model real stock price behavior. Jump can appear at a random time and these jumps can not be captured by the log normal distribution characteristic of the stock price in the Black Scholes model. To overcome the above shortcoming, several models have been proposed in the literature. Among these, the jump diffusion models introduced by Merton [23] and Kou [18] are of the most widely used models. Merton proposed a log-normally distributed process for the jump-amplitudes, whereas Kou suggested log-double-exponentially distributed process.

The valuation of option under jump diffusion process requires the solution of a partial integro-differential equation containing a non-local integral term. There are several numerical methods available in the literature to approximate the above equation. In [1], Almendral and Osterlee presented an implicit second order accurate time discretization with finite difference and finite element spatial discretization on uniform grid. Andersen et al. [2] proposed an unconditionally stable alternating direction implicit method for its solution. Song Wang et al. [33] developed a fitted finite volume method for jump diffusion process. Their method is based on fitted finite volume method spatial discretization and Crank Nicolson scheme for temporal discretization. More recently, Patidar et al. [24] developed an efficient method for pricing Merton jump diffusion option, combining the spectral domain decomposition method and the Laplace transform method. The scheme proposed by d Halluin et al. [9] required to use an iterative procedure to solve discrete equations. The main difficulty with implicit scheme is due to containing non-local integral term in governing equation, which leads to a dense discretization matrix where as fully explicit scheme imposed stability restriction on it. An approach based on implicit–explicit schemes in which integral term is treated explicitly has been proposed by YongHoon Kwon et al. [19] and Briani et al. [3]. More

* Corresponding author.
E-mail address: alpeshmath@gmail.com (A. Kumar).

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recently Tangman et al. [28] introduced a new scheme called exponential time integration (ETI) scheme to solve the PIDE. In ETI scheme, the time direction of PIDE is directly tackled by a ‘one step’ formula, which means temporal discretization is not required. Tangman et al. [28] used the central difference approach with ETI to provide very efficient and second order accurate result.

Recently, a new method based on radial basis function (RBF) for approximation of spatial derivative in option pricing equation is under going active research. Application of RBF in one dimension European and American options is given by Hon et al. [13,14]. Fasshauer et al. [10] solved American option pricing model using penalty method.

Golbabai et al. [12], developed an algorithm based on global collocation for jump diffusion process. Bhuruth et al. [25] proposed a radial basis function based differential quadrature rule for spatial discretization with exponential time integration to solve jump diffusion model. In more recent work, Chan et al. [4,6] used new RBF called cubic spline as basis function to solve PIDE, and show that their scheme is second order accurate.

It was recognized that standard approach to solving the radial basis function collocation problem has been ill conditioned due to use of collocation in global sense. Recently many strategies have been developed in the literature to avoid these problems, such as local RBF approach by Lee et al. [21], radial point interpolation method proposed by Liu et al. [22], Shu et al. [27] proposed a local radial basis function-based differential quadrature method and used it to solve two-dimensional incompressible Navier–Stokes equations. Tolstykh [30], Tolstykh and Shirobokov [31], Wright et al. [11,32] proposed radial basis function finite difference method, the idea is to use radial basis functions with a local collocation as in finite difference mode thereby reducing number of nodes and hence producing a sparse matrix. This technique is further extended by Sanyasiraju et al. [7,26] for convection diffusion type equations. However these methods have not been extended to solve partial integro differential equation yet. In the present work, we have extended the localization concept proposed by Wright and Fornberg, to solve jump diffusion models. The governing equations are discretized by a three level implicit and explicit time scheme followed by RBF based finite difference method.

The paper is organized as follows. In section 2, mathematical models for pricing option with jump diffusion process are given in terms of partial integro-differential equations and provide a brief review of both the Merton and Kou jump diffusion models. Section 3 deals with the construction of three time level implicit explicit scheme to discretize the jump diffusion model. The time semi discrete equation is coupled with radial basis function based finite difference method for spatial discretization. Section 4 provides extension of proposed method for pricing American option by utilizing concept of operator splitting method. In section 5, we give some numerical results for Merton and Kou model and a comparison of the accuracy of our solution with finite difference and finite element method for both American and European options. Finally the paper ends with some conclusive remarks in section 6.

2. The mathematical model

In this section, we give brief discussion about the mathematical model for option with jump diffusion process. Consider an asset with the asset price S, then the movement of stock price is modeled by the following stochastic differential equation

$$\frac{dS}{S} = (\nu - \kappa\lambda)d\tau + \sigma dZ + (\eta - 1)dq \tag{2.1}$$

where, ν is drift rate, τ as the time to maturity, σ represents the constant volatility, dZ is an increment of standard Gauss–Wiener process. The term λ is the intensity of the independent Poisson process dq with

$$dq = \begin{cases} 0 & \text{with probability } 1 - \lambda d\tau, \\ 1 & \text{with probability } \lambda d\tau. \end{cases}$$

The expected relative jump size $\mathbb{E}(\eta - 1)$ is denoted by κ, where $\mathbb{E}[\cdot]$ is the expectation operator and $\eta - 1$ is a impulse function producing jump from S to $S\eta$.

Let $V(S,\tau)$ represent the value of a contingent claim that depends on the underlying asset price S with current time τ. Then $V(S,\tau)$ satisfy following backward partial integro differential equation

$$\frac{\partial V}{\partial \tau} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda\kappa)S\frac{\partial V}{\partial S} - (r + \lambda)V + \lambda \int_0^\infty V(S\eta)g(\eta)d\eta = 0, \tag{2.2}$$

for $(S,\tau) \in (0,\infty) \times (0,T]$, where, r is risk free interest rate and $g(\eta)$ is probability density function of the jump with amplitude η with properties that $\forall\eta$, $g(\eta) \geq 0$ and $\int_0^\infty g(\eta)d\eta = 1$.

The value of V at the expiry date is given by,

$$V(S,T) = \mathcal{G}(S), \quad S \in (0,\infty), \tag{2.3}$$

where $\mathcal{G}(S)$ is the pay-off function for the option contract. Under Merton’s model $g(\eta)$ is given by the log-normal density