Applied Numerical Mathematics 109 (2016) 157–183

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

A quasi-static contact problem in thermoviscoelastic diffusion theory

M.I.M. Copetti [a], M. Aouadi [b,*]

[a] Laboratório de Análise Numérica e Astrofísica, Departamento de Matemática, Universidade Federal de Santa Maria, 97105-900, Santa Maria, RS, Brazil

[b] Ecole Nationale d'Ingénieurs de Bizerte, Université de Carthage, BP66, Campus Universitaire, Menzel Abderrahman 7035, Tunisia

ARTICLE INFO

Article history:
Received 26 September 2015
Received in revised form 25 June 2016
Accepted 29 June 2016
Available online 11 July 2016

Keywords:
Thermoviscoelastic
Diffusion
Contact
Existence
Exponential stability
Numerical analysis

ABSTRACT

The problem of thermoviscoelastic quasi-static contact between a rod and a rigid obstacle, when the diffusion effect is taken into account, is modeled and analyzed. The contact is modeled by the Signorini's condition and the stress–strain constitutive equation is of the Kelvin–Voigt type. In the quasi-static case, the governing equations correspond to the coupling of an elliptic and two parabolic equations. It poses some new mathematical difficulties due to the nonlinear boundary conditions. The existence of solutions is proved as the limit of solutions to a penalized problem. Moreover, we show that the weak solution converges to zero exponentially as time goes to infinity. Finally, we give some computational results where the influence of diffusion and viscosity are illustrated in contact.

1. Introduction

Thermoelastic contact problems arise naturally in many applications (see e.g., [8] and [14]). One of the most important are sliding systems such as brakes, clutches and seals. Also they are found in industrial processes and everyday life such as train wheels with the rails, a shoe with the floor, tectonic plates, the car's braking system, etc. In these situations two or more elastic materials are forced into contact with each other as a result of thermal expansion. Predicting the behavior of thermoelastic contacting bodies in such situations is of considerable applied importance. The anomalous behavior of some thermoelastic systems has been investigated, for example, by Barber et al. [7], Richmond and Huang [26] and Srinivasan and France [29].

In spite of the obvious applied importance of the subject, there are relatively few theoretical results about general problems of thermoelastic contact. At times, the mathematical models used to solve contact problems make various restrictive assumptions. In Duvaut and Lions [16], existence theorems were proved under the condition that there was no loss of contact. Assuming that the temperature field does not depend on the stress and displacement fields, the semi-coupled theory of thermoelasticity is obtained [1]. In this case the temperature distribution is first obtained and then used to determine the displacements.

In the last years a number of publications have appeared dealing with various models for the different aspects of quasi-static contact. These are processes where the applied forces vary slowly, and therefore, the system response is relatively slow, so that the inertial terms in the equations of motion can be neglected. This means that the propagation of waves is

* Corresponding author. Fax: +216 72 570 586.
E-mail addresses: mimcopetti@ufsm.br (M.I.M. Copetti), moncef_aouadi@yahoo.fr (M. Aouadi).

http://dx.doi.org/10.1016/j.apnum.2016.06.011
0168-9274/

neglected, i.e., the system is observed on a time scale much longer than that necessary for any waves to travel through it and to decay. Only recently, however, have the quasi-static and dynamic problems been considered. The reason lies in the considerable difficulties that the process of contact presents in the modeling and analysis because of the complicated surface phenomena involved. The one-dimensional quasi-static problem of thermoelastic contact was considered in a series of papers. The problem was formulated as a fully coupled variational inequality in [17] and the existence of a solution was established. A reformulation of the problem in [27] led to a decoupled heat equation with a nonlinear and nonlocal source term. The uniqueness of the solution was proved as well as the fact that the solution converges to a steady state. In both papers [17] and [27] the Dirichlet condition was assumed for the temperature at the contacting edge. Barber and others (see, e.g., [6] and references therein) found that the assumption of perfect thermal insulation when there is no contact and perfect thermal contact otherwise leads to the nonexistence of steady-state solutions. This leads to the introduction of a pressure dependent heat exchange condition. The more realistic heat exchange condition was considered in [28], where the existence of a solution was obtained from an abstract result involving perturbations of monotone operators.

Since contact is observed as a result of thermal expansions or contractions, contact thermoelastic problems were developed without considering the mass diffusion which has an important conduction effect. To the best of our knowledge, the first papers considering the mass diffusion conduction effect in thermoelastic and thermoviscoelastic contact problems are [3–5]. We may think that the theory of thermoelasticity is a good model to explain contact between deformable bodies in industrial process. In fact, the development of high technologies strongly affected the investigations in which the fields of temperature and mass diffusion in solids cannot be neglected. Diffusion can be defined as the random walk of a set of particles from regions of high concentration to regions of lower concentration. Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. At elevated and low temperatures, the processes of heat and mass transfer play an important role in many industrial applications.

Indeed, contact problems in thermoelasticity are not in general well understood and mathematical results are relatively rare. This lack of results is generally explained by the mathematical difficulties encountered in treating such problems. But we think that a part of these difficulties are caused by the absence of the mass diffusion effect in the framework of linearized thermoelasticity. A natural question is to know what happens when the mass diffusion effect is considered in thermoelastic contact problems. This question is not only interesting from a mechanical, mathematical and numerical viewpoint, but especially economically. In fact, an estimated 0.5 percent of the US GNP is lost (see, e.g., [25]) because of insufficient control of contact processes in machines, cars and mechanical equipment. By considering frictional losses and the possible damage caused by the diffusion conduction effect, we contribute to an accurate prediction of the evolution of contact processes and their control.

In this work we consider a general model for the quasi-static process of thermoviscoelastic contact between a deformable body and a rigid obstacle. The material is assumed to behave according to the Kelvin–Voigt constitutive law with added thermal and diffusion effects. The considered system consists of a parabolic equation of motion coupled with two parabolic equations of temperature and diffusion and so does not belong to any one of the standard categories of systems of equations. The derivation of such systems can be found in a number of papers (see e.g., [19–21]). We will show that a quasi-static, fully coupled problem of thermoviscoelastic contact, in one space dimension, possesses a weak solution. We prove that, the presence of viscoelastic term in the equations provides additional regularity, existence and exponential stability. This paper is an attempt to make the contact between deformable bodies a better understood process.

The paper is structured as follows. In Section 2, we present the mathematical model and we set up the function spaces in which we are going to work and specify the requirements on boundary and initial data. We then formulate the variational problem. In Section 3, we introduce a penalized problem and we use a Faedo–Galerkin method to turn the variational penalized problem into a finite dimensional problem. Then we get the necessary a priori estimates to pass to the limit and prove that the penalized problem admits only one solution. In Section 4, we pass to the limit in the penalized problem in order to prove that the variational formulation of the original problem admits only one solution. In Section 5 we show the exponential decay of the solution for both the penalized and the contact problem. In section 6, we describe the numerical method based on the finite element approximation with some results on the error bound and rate of convergence. Finally, in section 7, we show the results of some numerical simulations by the method described in [10], where the influence of diffusion and viscosity are illustrated in contact.

2. Basic equations and preliminaries

The theory of thermoviscoelastic diffusion of the Kelvin–Voigt type was introduced by Kubik (see [19–21]). Let us make a short presentation of the general three-dimensional theory. The evolution equations of the dynamic problem are:

(i) The equation of motion:

$$\rho \ddot{u}_i = \sigma_{ij,j}. \tag{1}$$

(ii) The stress–strain–temperature–diffusion relation:

$$\sigma_{ij} = 2(\mu e_{ij} + \mu' \dot{e}_{ij}) + \delta_{ij}(\lambda e_{kk} + \lambda' \dot{e}_{kk}) - \beta_1 \delta_{ij}(T - T_0) - \beta_2 \delta_{ij} C. \tag{2}$$