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A maximum-principle-satisfying finite volume compact-WENO scheme for traffic flow model on networks

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ABSTRACT

In this paper, we apply a maximum-principle-satisfying finite volume compact weighted scheme to numerical modeling traffic flow problems on networks. Road networks can be numerically model as a graph, whose edges are a finite number of roads that join at junctions. The evolution on each road is described by a scalar hyperbolic conservation law, and traffic distribution matrices are used to formulate coupling conditions at the network junctions. In order to achieve maximum-principle of the traffic density on each road, the maximum-principle-satisfying polynomial rescaling limiter is adopted. Numerical results for road networks with rich solution structures are presented in this work and indicate that the finite volume compact weighted scheme produces essentially non-oscillatory, maximum principle preserving and high resolution solutions.

1. Introduction

Microscopic model of traffic flow attempts to describe the evolution of the vehicular traffic density by using partial differential equations. In the 1950s, Lighthill and Whitham [18] and Richards [24] independently proposed a simple first order fluid approximation of traffic flow dynamics to describe the evolution of traffic flow on a single road. In recent years, many fluid-dynamic models for traffic flow based on this classical Lighthill–Whitham–Richards (LWR) model were developed. Wong and Wong [25] proposed a multi-class model with heterogeneous drivers. Daganzo proposed a macroscopic behavioral theory of traffic dynamics for homogeneous, multi-lane freeways in [8]. Recently, there are a number of different models have been proposed for traffic flow on road networks [6,14,9]. On each road, the traffic network is described by using partial differential equations, specifically conservation laws, at the macroscopic scale.

The classical LWR partial differential equation [18,24] is a scalar hyperbolic conservation law and can be numerically solved by a variety of numerical methods, such as first-order Lax–Friedrichs finite difference scheme [22] and Godunov's scheme [7,16]. The first-order Lax–Friedrichs finite difference scheme is also used to solve multi-class models. The model takes into account the distribution of heterogeneous drivers characterized by their choice of speeds in a traffic stream [25]. Numerical solutions to traffic flow problems on road networks are provided in [1] by using classical first order Godunov scheme and kinetic schemes. For these low order schemes, we should use refined mesh to diminish the effect of numerical diffusion near discontinuities. In recent years, a number of high-order numerical schemes were proposed for solving continuum traffic flow models. Nevertheless, it is difficult to design efficient and high order scheme for the LWR model because

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of the inherent presence of discontinuities in the solution. Zhang et al. proposed a high-order weighted essentially non-oscillatory scheme for solving a multi-class LWR model in [28], and the numerical results indicate that the WENO method is more efficient than the low-order Lax–Friedrichs scheme. In [29], the WENO method is also applied to solve a multi-class traffic flow model with spatially varying fluxes for an inhomogeneous highway. Liu et al. applied WENO approach to model the asymmetry in traffic flow [20]. Recently, Canic et al. proposed a bound-preserving Runge–Kutta discontinuous Galerkin method with arbitrary high-order accuracy for numerical simulation of traffic flow problems on networks in [3], where an arbitrary high order bound preserving limiter is applied to the Runge–Kutta discontinuous Galerkin method to preserve the physical bounds on the network solutions.

In the past few years, a variety of high order schemes which are non-oscillatory for discontinuous solutions have been proposed for scalar hyperbolic conservation equation, such as WENO schemes [19,15] and TVB Runge–Kutta discontinuous Galerkin methods [5]. It is well known that the entropy solution of conservation laws should satisfy a strict maximum-principle. Recently, Zhang and Shu proposed uniformly high order accurate schemes that satisfy a strict maximum-principle based on the finite volume WENO scheme for scalar conservation laws [27]. However, non-compact WENO schemes often suffer from relatively high numerical dissipation [21], poor spectral resolution and increasingly wide stencils with increasing order of accuracy. Compared with non-compact WENO schemes, classical compact schemes [17] have significant higher spectral resolutions with narrower stencils by using global grids. However, these classical linear compact schemes generate spurious oscillations near or across discontinuities. Pirozzoli proposed a hybrid compact WENO scheme in [23], where a fifth-order compact upwind algorithm in conservation form is used for solving the smooth part of the flow field, which is coupled with WENO scheme to capture the discontinuities. In [10], Ghosh and Baeder developed compact reconstruction finite difference WENO schemes, where lower order biased compact stencils were used to formulate higher order upwind interpolation with optimal weights. These compact schemes introduced above are all based on a finite difference framework. However, in order to satisfy the governing laws of the fluid physics, finite volume approaches which are inherently conservative are usually adopted. In [13], Guo et al. employed the idea that is described in [10], and proposed a finite volume compact scheme for solving scalar hyperbolic conservation laws where maximum preserving limiters [26,27] are applied to the FVCW scheme. A similar idea with positivity preserving limiter [27] for one dimensional compressible Euler system has been explored in [12].

In this paper, we propose to apply the maximum-principle-satisfying finite volume compact-WENO scheme [13] for simulating hyperbolic road network problems. To keep the essentially non-oscillatory properties for capturing shocks, lower order compact stencils are combined with WENO weights to yield a fifth-order upwind compact interpolation. By applying a polynomial scaling limiter [26,27] to the FVCW scheme at each stage of an explicit Runge–Kutta method, the scheme preserves the upper and lower bounds of numerical solutions. A number of numerical examples for both benchmark problems [1] and challenging cases with rich solution structures presented in [3] are numerically computed by using the compact scheme in finite volume framework. By comparing with numerical results presented in [3], the proposed compact schemes produce spectral-like resolution numerical solutions without spurious oscillations. A challenging case with very rich solution structures is set to test the maximum principle preserving property and the high resolution of the FVCW scheme. The numerical results show that the minimum values of numerical density are negative without limiters. We can also observe that the present scheme produces higher resolution numerical solutions than those obtained with the classical fifth-order WENO scheme without spurious oscillations. Many other numerical results will show maximum principle preserving and high resolution property of the proposed approach.

The rest of this paper is organized as follows. In Section 2, the class of traffic models on networks under consideration is reviewed. The finite volume compact-WENO scheme for the governing equation for the first-order LWR model in one dimension and the maximum principle satisfying limiter are introduced in Section 3. In Section 4, we will show the numerical results for some network problems. Concluding remarks are given in Section 5.

2. Macroscopic traffic models

In a single road, the nonlinear LWR model based on the conservation of cars is described by a scalar hyperbolic conservation law as follows [18,24]

$$\partial_t \rho + \partial_x f(\rho) = 0, \tag{1}$$

where $\rho = \rho(t,x) \in [0, \rho_{max}]$ is the density of cars at time t, with $\rho_{max} > 0$ is the maximum density of cars. $f(\rho) = \rho v$ is the flux, one can assume that v is a given smoothing decreasing function of density, depending only on the density. The usual assumptions on f is that $f(0) = f(\rho_{max}) = 0$ and flux f is strictly concave.

In this paper, we consider a road network introduced in [2,6], which is a finite number of roads $[a_i, b_i]$ that meet at some junctions. Consider a junction J with incoming roads I_i, where $i = 1, \cdots, n$ and outgoing roads I_j, $j = n+1, \cdots, n+m$. The choice of the outgoing road is prescribed by traffic distribution matrix [2,6]

$$A = \begin{bmatrix} \alpha_{n+1,1} & \cdots & \alpha_{n+1,n} \\ \cdots & \cdots & \cdots \\ \alpha_{n+m,1} & \cdots & \alpha_{n+m,n} \end{bmatrix},$$