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A HLL-type Riemann solver for two-phase flow with surface forces and phase transitions

Veronika Schleper

University of Stuttgart, Institute for Applied Analysis and Numerical Simulation, Pfaffenwaldring 57, D-70569 Stuttgart, Germany

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ABSTRACT

We present a new HLL-type approximate Riemann solver for a compressible two-phase flow model with phase transition and surface forces such as surface tension or electric forces. The solver is obtained following the main ideas of the HLL-approach. Due to the nonlinearity in the kinetic relation driving the phase transition, this solver involves the solution of one single nonlinear equation, contrary to a single-phase HLL-type solver, where no nonlinear equations have to be solved. We present some illustrative numerical examples to show the performance and accuracy of the new solver, comparing it to the relaxation Riemann solver from the literature as well as to the two-shock approximation, which is the most accurate interface Riemann solver currently available for the present situation, but suffers from high computational costs.

1. Introduction

Compressible two-phase flow simulations require special attention when describing the interaction of the fluids at material interfaces, which must be modeled to reproduce the underlying physics in a correct way. A direct numerical simulation of the two-fluid flow with a sharp interface representation therefore typically consists of three key elements. First, an accurate method to describe the temporal movement of the material interface is needed. The second key ingredient is a good thermodynamical model to describe the properties of each single fluid in the range of interest. Third, we have to resolve the physics at the fluidic interface using a model that is able to reproduce the interaction between the fluids. The first two elements are based on a conservative flow solver that is capable to cope with arbitrary equations of state.

In this paper, we focus on the last of these three key points. The simplest way to treat such interfaces is the use of a standard Ghost-Fluid Method without any special micro model at the interface as proposed by Fedkiw et al. in [8]. However, this approach does not take the different physical properties of the two fluids into account, given by dissimilar equations of state, and as a consequence fails to reproduce the physically correct interface behavior, see e.g. [14]. Therefore, special care is needed to assign the values of the ghost states. The situation is even worse when external forces act on the interface, resulting in a significant change of the interface behavior. Examples for such surface forces are for instance surface tension or electric forces acting on charged droplets in electric fields.

One possible way to achieve an accurate interface behavior is to use exact Riemann solvers at the phase boundaries to predict the correct interface states. Even though this technique will yield the most accurate interface movement, it has some important drawbacks. First of all, the interface Riemann solver has to be derived for every new equation of state separately and therefore makes the simulation code very inflexible for simulations with physically correct equations of state. Second,

E-mail address: veronika.schleper@mathematik.uni-stuttgart.de.

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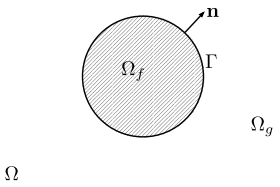

Fig. 1. Illustration of the different domains for the compressible multiphase model. The fluids f and g are separated by the sharp interface Γ.

the exact solution to the interface Riemann problem can, in general, be found only by an iterative procedure. For complex equations of state, the high number of function evaluations during the iterative process as well as convergence issues make this approach very unattractive. Alternative methods using two shock or two rarefaction approximations of the true Riemann solution can reduce the computational costs, but cannot overcome the convergence issues and the lack of flexibility.

The use of approximate Riemann solvers is therefore very attractive, as they usually yield a solution without iterative procedure and only few evaluations of the equation of state in the case of two-fluid flows. Due to possibly non-linear kinetic relations for phase transitions, iterative solvers will be needed in this case also for approximate Riemann solvers. However, the computational complexity as well as the dimension of the nonlinear equations to solve can be drastically reduced by approximate solvers. One of the most commonly used approximate Riemann solvers for fluid dynamics is the class of HLL-solvers developed in [9,17] and extended to two-fluid flow without phase transitions in [11].

Following the solution structure of the two-fluid HLL-solver, we derive the HLLP-solver for phase transition problems with non-linear kinetic relations. This solver is shown to preserve many properties of the HLL-class, such as the positivity preserving property and exact resolution of equilibrium interfaces (a property that corresponds to the exact resolution of isolated contact discontinuities in the case of the single-phase HLLC-solver). As stated before, the nonlinear structure of the kinetic relation forces us to use an iterative solution procedure that is reduced from seven unknowns to one.

In the present publication, we extend the results of [7] to the case of nonlinear interface relations that allow phase transitions. Contrary to the approximate Riemann solvers in [7], which are based on linearization or relaxation approaches, the new approximate solver is based on the HLL-approach. This allows to prove that the solutions constructed by the HLL-type approximate Riemann solver remain in the respective domains of definition of the respective phases and as a consequence are also positivity preserving. Furthermore, we show that the new solver yields accurate results, different from the relaxation approach developed in [16], where severe discrepancies between reference solution and approximation can be seen, while the computational costs of both solvers are comparable.

The above approximate Riemann solvers are designed for use in a sharp interface method for the simulation of two phase flows in the presence of external forces acting *on the surface* of one of the two fluids. This method fits into the framework of the heterogeneous multiscale method [4] and is already used in similar context in [12,5]. The presented method is a very powerful tool for the simulation of various physical processes, as it can be easily adapted to catch all relevant physical phenomena at the interface by simply exchanging the Riemann solver at the interface.

Compared to methods with diffuse interface representation as volume-of-fluid [10] or phase field models, see [2] for a review, the presented sharp interface model however has the advantage that surface forces can be applied directly at the fluid–fluid interface and physically correct formulations are therefore easy to obtain. Contrary to that, new phase field models have to be implemented in most cases when one wishes to include new physical aspects, as the coupling of interface motion and fluid flow is not local.

The paper is organized as follows. Section 2 introduces the basic model and the interface jump conditions for various two-phase flow situations. The derivation of HLL-type Riemann solvers can be found in Section 3. Finally, we present some illustrative numerical examples in Section 4, showing the accuracy of the new approximate Riemann solver compared to a relaxation approach from the literature.

2. A compressible two-phase model with surface forces and phase transitions

As stated above, we use a heterogeneous multiscale approach to model the motion of compressible multiphase flows. To this end, we assume that the two fluids f in Ω_f and g in Ω_g are separated by a sharp interface Γ, such that $\Omega = \Omega_f \cup \Gamma \cup \Omega_g$, see also Fig. 1.

The motion of each fluid $f \in \Omega_f$ and $g \in \Omega_g$ is governed by the isothermal Euler equations in Ω_i

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{v}) = 0 \tag{1a}$$

$$\partial_t (\rho \boldsymbol{v}) + \nabla \cdot (\boldsymbol{v} \times (\rho \boldsymbol{v})) + \nabla p_i(\rho) = 0 \tag{1b}$$

where ρ is the mass density and $\boldsymbol{v} = (v_1, v_2, v_3)^T$ the velocity vector. For $i \in \{f, g\}$, $p_i(\rho)$ denotes the pressure and is defined by an equation of state individually for each fluid in the case of two-fluid flow. The pressure laws $p_i(\rho)$ may also be given by the respective branch of a non-convex pressure law in the case of phase transitions. More details on the choice of such equations of state are given below.