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An explicit-extended penalty Galerkin method for solving an incompressible two-phase flow

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ABSTRACT

In this paper, we present a stabilized explicit-extended penalty Galerkin method based on the implicit pressure and explicit saturation method to find the global solution for the two-phase flow in porous media at each time step. The bubble functions are employed as basis of the spatial dimensions for the extended penalty Galerkin method. The forward Euler method is applied to the temporal discretization. Since the accuracy of numerical simulations flow through porous media depends on the modeling of the injection and production well, we propose a new well model for the presented method. The details of the stability analysis for the proposed method are provided and suitable values of the penalty term and time steps are calculated. The efficiency of the method is illustrated by simulations of a waterflood in a heterogeneous oil reservoir. Comparisons are made with available literature which show the efficiency and accuracy of the proposed method.

1. Introduction

The waterflooding technique is used in the secondary oil recovery methods. In this technique the water is injected into some wells to maintain the field oil pressure and to push the oil to production wells [20]. Modeling of the simultaneous flow of two fluids, so-called two-phase flow, plays an important role in the waterflooding (see, e.g., [5,11]).

Several locally conservative methods, such as finite difference [17], finite element [13] and finite volume [16] methods have been applied for the spatial discretization of two-phase flow equations. Unlike local methods, spectral methods give a global representation of the approximate solution. One of these methods is the penalty Galerkin method. In the penalty Galerkin method, we seek for a polynomial solution whose residual is orthogonal to the polynomial space and the boundary conditions are enforced to use a penalty method. The process employed in this method is exactly the same as applying the discontinuous Galerkin (DG) method on a subdomain [12]. Application of DG methods to incompressible two-phase flow may be found e.g. in Refs. [3,2,10].

For numerical simulation of a reservoir, it is necessary to use grid cells whose horizontal dimensions are much larger than the diameter of a wellbore. If so, the oil pressure computed for a cell containing a well is greatly different from the bottom hole pressure (BHP) of the well [8,21].

This paper aims to provide a stabilized explicit-extended penalty Galerkin method. In fact we present an extended penalty Galerkin method (EPG) for the spatial discretization and use the forward Euler method for the temporal discretiza-

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Nomenclature

μ_α $(\alpha = o, w)$	viscosity	cp	P_{bhw}, P_{bho}	bottom hole pressure	psi
ν	outward unit normal	–	P_{cow}	capillary pressure	psi
ϕ	porosity	–	r_e	wellbore radius	ft
ρ_α, $\alpha = o, w$	density	lbm/ft^3	S_α $(\alpha = o, w)$	saturations	–
g	32.174	ft/s^2	t	time	day
K	absolute permeability	md	x, y	length, width	ft
k_α $(\alpha = o, w)$	relative permeability	–	Z	depth	ft
P_α $(\alpha = o, w)$	pressure	psi			

tion. The proposed method is simple to set up and efficient to implement, and requires less computational costs than other methods available such as the mixed finite element method [6] and finite volume method [16]. Since local methods are generally sensitive to the grid size for obtaining an accurate approximate solution, one needs to increase the mesh size and perform a heavy computation. We find a polynomial solution for both oil pressure and water saturation. Also, we propose a novel well model to simulate the behavior of the injection and production well. The oil pressure in the wells is approximately the same as BHP by using this novel model. We show that the proposed method is stable and find the suitable values of the penalty term and the time steps for forward Euler method.

2. Governing equations

The governing equations describing two-phase (oil and water) incompressible, immiscible flow in a heterogeneous porous media are given by [11]:

$$\frac{\partial}{\partial t}(\phi S_\alpha \rho_\alpha) + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0, \qquad \alpha = o, w, \tag{1}$$

where the volumetric velocity $\mathbf{u}_\alpha = -K\dfrac{k_{r\alpha}(S_\alpha)}{\mu_\alpha}(\nabla P_\alpha - \rho_\alpha g \nabla Z)$ is given by Darcy law. It is assumed that there is no flow boundary condition i.e. $\mathbf{u}_\alpha \cdot \nu = 0$ for $\alpha = o, w$ where ν is the outward unit normal to the porous media.

By neglecting gravitational effect and removing the density from both sides of equations, since fluids are immiscible, Eqs. (1) can be described as follows:

$$\frac{\partial}{\partial t}(\phi S_o) - \nabla \cdot (K\frac{k_{ro}(S_o)}{\mu_o}\nabla P_o) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\phi S_w) - \nabla \cdot (K\frac{k_{rw}(S_w)}{\mu_w}\nabla P_w) = 0. \tag{3}$$

with boundary conditions

$$-K\frac{k_{ro}(S_o)}{\mu_o}(\nabla P_o \cdot \nu) = 0, \qquad -K\frac{k_{rw}(S_w)}{\mu_w}(\nabla P_w \cdot \nu) = 0. \tag{4}$$

In addition, the customary property for the water saturations is $S_o + S_w = 1$ and the two pressures are related by the capillary pressure function $P_{cow}(S_w) = P_o - P_w$.

Adding Eqs. (2) and (3), employing the water saturation property and the capillary pressure function leads to the following oil pressure equation [7]:

$$-\nabla \cdot (K\lambda \nabla P_o) + \nabla \cdot (K\lambda_w \frac{\partial P_{cow}}{\partial S_w}\nabla S_w) = 0, \tag{5}$$

where $\lambda(S_w) = \lambda_o(S_w) + \lambda_w(S_w)$ is the total mobility in which $\lambda_\alpha(S_\alpha) = \frac{k_\alpha(S_\alpha)}{\mu_\alpha}$ for $\alpha = o, w$ are the phase mobilities. Since the total mobility is always positive, the oil pressure equation (5) is elliptic.

Notice that substituting the total velocity $\mathbf{u} = \mathbf{u}_o + \mathbf{u}_w$ into Eq. (3), the water saturation equation can be expressed by [7]:

$$\phi\frac{\partial S_w}{\partial t} + \nabla \cdot \{K\frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o}\frac{dP_{cow}}{dS_w}\nabla S_w + \frac{\lambda_w}{\lambda_w + \lambda_o}\mathbf{u}\} = 0, \tag{6}$$

which is a parabolic equation in S_w. In general, $-K\dfrac{\lambda_w \lambda_o}{\lambda_w + \lambda_o}\dfrac{dP_{cow}}{dS_w}$ is non-negative, so the water saturation equation (6) is a parabolic equation and it is a nonlinear equation, since the λ_w and λ_o depend on S_w. We use implicit pressure explicit