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Numerical modeling of sediment transport applied to coastal morphodynamics

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ABSTRACT

A bed-load sediment transport model is used to describe realistic cases of the morphodynamics in coastal areas. The hydrodynamic equations are based on the well-known, two-dimensional depth-averaged non-linear shallow water equations, with bathymetry forces and friction, which are subsequently coupled to the Exner equation to describe the morphological evolution. Different forms of the bed-load transport flux are considered in the Exner equation and certain relations between them are established. The numerical model is expressed in a fully-coupled form where a single system of equations is solved by a high-resolution two-dimensional finite volume scheme of the relaxation type. The relaxation is performed by classical models where neither approximate Riemann solvers nor characteristic decompositions are needed. The overall numerical scheme is validated in benchmark problems, and for a realistic application of a coastal area on the northern side of the island of Crete. The validity of these results is established by comparisons made with the well-known MIKE Software by DHI Group.

1. Introduction

Quantifying the interaction between sediment transport and water flow is crucial for a wide range of phenomena such as near-shore and river morphodynamic evolution, river management, and river adjustment after the installation of hydraulic structures. A typical example is the morphodynamics of coastal areas, where sediment transport has a leading role in understanding the formation and evolution of the beach zone. To progress on quantifying such interactions, it becomes necessary to develop numerical models that accurately simulate the fluid flow over a movable bed. Numerical modeling of free surface flows with load transport over erodible bed in realistic situations involves transient flow and movable boundaries.

In the present work, we consider two-dimensional (2D) bed-load transport simulations based on the depth-averaged non-linear shallow water equations (NSW) and an additional continuity equation modeling the morphodynamic component [15]. The 2D NSW equations with variable topography and friction represent depth-averaged mass and momentum conservation and can be obtained by depth-integrating the Navier–Stokes equations. Neglecting wind effects, Coriolis forces and diffusion of momentum due to viscosity and turbulence, they form a system of equations written in their conservative (balance law) form as:

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$$\widetilde{\mathbf{U}}_t + \widetilde{\mathbf{F}}(\widetilde{\mathbf{U}})_x + \widetilde{\mathbf{G}}(\widetilde{\mathbf{U}})_y = \widetilde{\mathbf{S}}(\widetilde{\mathbf{U}}, x, y); \quad (x, y) \in \mathbb{R}^2, \ t > 0,$$
$$\widetilde{\mathbf{U}}(x, y, 0) = \widetilde{\mathbf{U}}_0(x, y); \quad (x, y) \in \mathbb{R}^2 \tag{1}$$

where $\widetilde{\mathbf{U}} = (h, hu, hv)^{\mathrm{T}}$ are the conserved variables with h representing the water depth, $(u, v)^{\mathrm{T}}$ the depth averaged components of the velocity vector $\mathbf{u}$ along the (x, y) coordinates, respectively. The subscripts in (1) denote partial derivatives. We also denote as $\mathbf{Q} = (Q^x, Q^y) = h\mathbf{u}$ the conserved specific discharges. The fluxes of these variables are given by:

$$\widetilde{\mathbf{F}}(\widetilde{\mathbf{U}}) = \left(hu, hu^2 + \frac{1}{2}gh^2, huv\right)^{\mathrm{T}}, \qquad \widetilde{\mathbf{G}}(\widetilde{\mathbf{U}}) = \left(hv, huv, hv^2 + \frac{1}{2}gh^2\right)^{\mathrm{T}}, \tag{2}$$

where g is the gravitational acceleration. The source terms of the system are

$$\widetilde{\mathbf{S}} = \left(0, \ -ghB_x - \frac{\tau_b^x}{\rho}, \ -ghB_y - \frac{\tau_b^y}{\rho}\right)^{\mathrm{T}}, \tag{3}$$

with ρ the water density, $B = B(x, y)$ the bed level and $\mathbf{T}_b = (\tau_b^x, \tau_b^y)^{\mathrm{T}}$ the bed shear stress due to steady flow, written in this work in terms of the Manning coefficient, n_m and is expressed as:

$$\frac{\tau_b^x}{\rho} = ghS_f^x, \quad S_f^x = \frac{n_m^2 u \|\mathbf{u}\|}{h^{4/3}}; \tag{4}$$
$$\frac{\tau_b^y}{\rho} = ghS_f^y, \quad S_f^y = \frac{n_m^2 v \|\mathbf{u}\|}{h^{4/3}}. \tag{5}$$

Sediment dynamics can be based on the formulation of a sediment continuity equation which states that the time variation of the bed level, in a certain volume, is due to the net variation of the solid transport through the boundaries of the volume, whose mathematical expression is the Exner equation. In the present work, and neglecting sediment material entering/leaving the volume by suspension/deposition, the 2D Exner equation is given as:

$$B_t + \xi \left(q_b^x\right)_x + \xi \left(q_b^y\right)_y = 0, \tag{6}$$

where now $B = B(x, y, t)$, $\xi = (1 - \sigma)^{-1}$, with $\sigma \in (0, 1)$ being the porosity of the bed material, which depends on the type of the sediment, $\mathbf{q}_b = (q_b^x, q_b^y)^{\mathrm{T}}$ the sediment transport discharge (flux), along the (x, y) coordinates.

The definition of the solid transport discharge $\mathbf{q}_b$ for granular and non-cohesive sediment can be seen as a deterministic problem or a probabilistic one. Different estimations have been obtained by empirical methods supported by experimentation. Even if these formulae are usually obtained for stationary flows in rivers, they can also be applied to tidal or coastal flows, effected by waves and currents, because the time response of the sediment is small in comparison with the period of the waves or tides. As such, many different formulas for modeling solid transport discharge are available in the literature, with some of the most applied being the Grass [23], Meyer-Peter and Müller [30] and the van Rijn [46] one, see also [11] and [34] for a review.

In practice, two different approaches are available to solve the system of Eqs. (1) and (6), we refer to [10] for a detailed overview. The first one is the uncoupled approach, where the hydrodynamic equations are solved first, to calculate the full evolution of the flow field, and then the Exner equation is solved to model the morphodynamic response. This approach is mainly justified by the different time scales that usually characterize the water flow and sediment transport and can be successful when dealing with slow or quasi-steady flows, involving slowly varying bed-load over a long time. The second approach relies on the full coupling of the equations within each time step. This approach is more suitable for stronger interactions between the water flow and the movement of sediment and for transcritical flows, producing more stable and robust results [10,27,47,3,5,14]. However, a relevant drawback for the numerical point of view, of this approach is that a conservative formulation is not available, so that non-conservative formulations are usually adopted, see, e.g., [27,11,20,12,5,7,43], amongst others.

A large amount of work has been done in the last decade to develop numerical methods for the solution of the coupled system using the finite volume approach producing accurate results, we refer to [26,27,11,12,34,3,4,9,44,7,43], among others. In most of these articles, Roe-type schemes where developed for the coupled (non-conservative) system which require the generation of appropriate flux Jacobian matrices for the whole system, thus requiring certain mathematical properties in the definition of the sediment transport formula. Further, adequate differentiations of the involved fluxes have to be performed as to produce the Roe average values and the average wave-speed estimators. To this end, the eigenvalues of these Jacobian matrices have to be computed, or derive an approximation of them [44]. However, a difficulty of the coupled system is to calculate these eigenvalues, as no simple analytical expressions exist, contrarily to that of the non-linear shallow water equations. This can be tedious and computationally demanding, especially for 2D computations. The complexity can be increased even more for high resolution schemes, where the unknowns are represented with higher-order reconstructions.

The present work applies and extends the work presented in [20] where a non-oscillatory high-resolution relaxation-type scheme was developed to solve different formulations of the coupled system using the Grass formula in the Exner