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A fitted finite element method for the numerical approximation of void electro-stress migration

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A B S T R A C T

Microelectronic circuits usually contain small voids or cracks, and if those defects are large enough to sever the line, they cause an open circuit. A fully practical finite element method for the temporal analysis of the migration of voids in the presence of surface diffusion, electric loading and elastic stress is presented. We simulate a bulk–interface coupled system, with a moving interface governed by a fourth-order geometric evolution equation and a bulk where the electric potential and the displacement field are computed. The method presented here follows a *fitted* approach, since the interface grid is part of the boundary of the bulk grid. A detailed analysis, in terms of experimental order of convergence (when the exact solution to the free boundary problem is known) and coupling operations (e.g., smoothing/remeshing of the grids, intersection between elements of the two grids), is carried out. A comparison with a previously introduced *unfitted* approach (where the two grids are totally independent) is also performed, along with several numerical simulations in order to test the accuracy of the methods.

1. Introduction

Microelectronic circuits contain thin lines of aluminium alloy, that make electric contact between neighbouring devices possible. These lines are passivated with a layer of oxide at large temperatures, and during the cooling process large stresses are induced. As the dimensions of microchips are reduced further and further, and since interconnects always contain small voids or cracks, it is of great interest to investigate the physical mechanisms that impede such a reduction, due to mechanical failures in the lines induced by the motion of the cracks. The problem analysed in this paper involves the evolution over time of voids in a conducting metal line where three different contributions to the drift of the voids are present: the surface tension, the electric field and the elastic energy. This phenomenon is known as *electro-stress migration*; for further details see, e.g., [36,14,3], and the references therein.

As the height of interconnect lines is much smaller than the dimensions of the base, voids generally fully penetrate the conducting material. Hence it is common to consider a two dimensional model for void electro-stress migration, and this is the approach that we are going to pursue in this paper. In addition, for ease of exposition, we assume that the interconnect line is given by a rectangular solid. The electric field is induced in the line by prescribing the voltage on its vertical boundaries, while the displacement field is induced by prescribing the stresses on its four boundaries.

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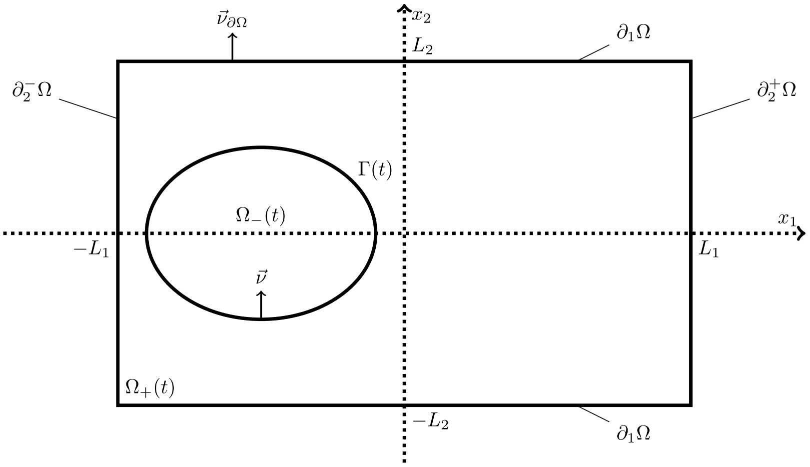

Fig. 1. The domain Ω and the void with its boundary $\Gamma(t)$.

In this paper, based on our previous work in [31], we introduce a novel front-tracking, *fitted* finite element method for the approximation of void electro-stress migration. The main difference to the approximation presented in [31] is that here we consider the fitted approach, which means that the interface mesh is always part of the boundary of the bulk grid. Moreover, we also include the effect of stress-migration into the model. As an aside we note that our method inherits the good interface mesh properties from the approximation in [31]. In particular, the vertices on the discrete interface equidistribute asymptotically so that no reparameterisation of the discrete interface is necessary in practice.

The paper is organised as follows. In Section 2 we give a mathematical description of the problem of void electro-stress migration that we are interested in. We also give a brief overview of the different numerical methods applicable to this problem. In addition, we highlight the differences between the fitted approach presented in this paper and the unfitted approach previously introduced by the authors in [31]. Section 3 contains a detailed description of our proposed finite element approximation. In Section 4 we discuss possible solution methods of the algebraic system of equations arising at each time level. In addition, we present details on the bulk mesh smoothing strategy. Finally, in Section 5 we perform a convergence experiment for a test case in which the exact solution is known, and we present various other examples of the application of our numerical method.

2. Problem formulation

For the formulation of the governing equations we closely follow the presentation in [31], see also [3]. Let $\Omega = (-L_1, L_1) \times (-L_2, L_2)$, where $L_1, L_2 > 0$, be the domain that contains the conductor. We denote the boundary of Ω with $\partial\Omega$. At any time $t \in [0, T]$, let $\Gamma(t) \subset \Omega$ be the boundary of the void $\Omega_-(t)$ inside the conductor Ω. Then $\Gamma(t) = \partial\Omega_-(t)$ and $\Omega_+(t) := \Omega \setminus \overline{\Omega_-(t)}$ denotes the conducting region (see Fig. 1). Now the evolution of the interface $\Gamma(t)$, which represents the void boundary, is given by

$$\mathcal{V} = -\alpha_1\, \varkappa_{ss} + \alpha_2\, \phi_{ss} + \alpha_3\, (E\,(\vec u))_{ss}, \tag{1}$$

where $\mathcal{V}$ represents the velocity of $\Gamma(t)$ in the direction $\vec\nu$ (the unit normal to $\Gamma(t)$ pointing into $\Omega_-(t)$), s is the arc-length of the curve, $\varkappa$ is the curvature of $\Gamma(t)$ (positive when $\Omega_-(t)$ is convex). In particular, it holds that

$$\vec x_{ss} = \varkappa\,\vec\nu\,, \tag{2}$$

where $\vec x$ is a suitable parameterisation of $\Gamma(t)$, i.e. $\Gamma(t) = \vec x\,(I, t)$, with $I = \mathbb{R}/\mathbb{Z}$ denoting the "periodic" interval $[0, 1]$.

The second contribution on the right-hand side of (1) is given by the electric potential $\phi(t)$, which satisfies a Laplace equation in $\Omega_+(t)$, i.e.:

$$\triangle\,\phi = 0 \quad \text{in}\ \Omega_+(t)\,, \qquad \frac{\partial\phi}{\partial\vec\nu} = 0 \quad \text{on}\ \Gamma(t)\,, \tag{3a}$$

$$\frac{\partial\phi}{\partial\vec\nu_{\partial\Omega}} = 0 \quad \text{on}\ \partial_1\Omega\,, \quad \phi = g^{\pm} \quad \text{on}\ \partial_2^{\pm}\Omega\,, \tag{3b}$$

where $\vec\nu_{\partial\Omega}$ is the outer normal to $\partial\Omega$. In (3b), $g^{\pm} := \pm L_1$ denotes the Dirichlet boundary condition on parts of $\partial\Omega$, where $\partial\Omega = \partial_1\Omega \cup \partial_2\Omega$, with $\partial_1\Omega \cap \partial_2\Omega = \emptyset$ and

$$\partial_2\Omega = \partial_2^-\Omega \cup \partial_2^+\Omega \quad \text{with} \quad \partial_2^{\pm}\Omega := \{\pm L_1\} \times [-L_2, L_2].$$

The Dirichlet boundary conditions in (3b) model a uniform parallel electric field, $\phi \approx x_1$ as $L_1 \to \infty$.