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Model order reduction using singularly perturbed systems

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ABSTRACT

Nowadays electronic circuits comprise about a hundred million components on slightly more than one square centimeter. The model order reduction (MOR) techniques are among the most powerful tools to conquer this complexity and scale, although the nonlinear MOR is still an open field of research. On the one hand, the MOR techniques are well developed for linear ordinary differential equations (ODEs). On the other hand, we deal with differential algebraic equations (DAEs), which result from models based on network approaches. There are the direct and the indirect strategy to convert a DAE into an ODE. We apply the direct approach, where an artificial parameter is introduced in the linear system of DAEs. This results in a singular perturbed problem. On compact domains, uniform convergence of the transfer function of the regularized system towards the transfer function of the system of DAEs is proved in the general linear case. This convergence is for the transfer functions of the full model. We apply and investigate two different ways of MOR techniques in this context. We have two test examples, which are both TL models.

1. Introduction

The tendency to analyze and design systems of ever increasing complexity is becoming more and more a dominating factor in progress of chip design. Along with this tendency, the complexity of the mathematical models increases both in structure and dimension. Complex models are more difficult to analyze, and it is also harder to develop control algorithms. Therefore model order reduction (MOR) is of utmost importance, see [5,7,28,30,33]. For the linear case, quite a number of approaches are well-established and resolve large systems of ordinary differential equations (ODEs) efficiently, see [1,5]. Extensions to parametric problems can be found in [11,13,18,19,14], nonlinear problems can be approximated by (piecewise) linearization, see [8,9,22]. Other techniques are proper orthogonal decomposition (POD) [25], discrete empirical interpolation (DEIM) [10] or use of bilinear approximations on reformulated ODEs [6].

We want to generalize according techniques to the case of linear systems of differential algebraic equations (DAEs). On the one hand, a high-index DAE problem can be converted into a low-index system by analytic differentiations, see [3]. A transformation to index zero yields an equivalent system of ODEs. On the other hand, a regularization is directly feasible

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in case of semi-explicit systems of DAEs. Thereby, we obtain a singularly perturbed problem of ODEs with an artificial parameter ε, which approximates the original DAEs. Thus according MOR techniques can be applied to the ODE system. An MOR approach for DAEs is achieved by considering the limit $\varepsilon \to 0$. This approach is different from [34]. Our approach allows for a broader class of methods to be applied.

In [21] we considered the regularization for semi-explicit systems of DAEs via an ε-embedding, i.e., the direct approach. The MOR techniques apply a transfer function defined in frequency domain. We proved the convergence of the transfer function of the regularized system (not yet reduced) to the transfer function of the original system of DAEs. Our approach also allows for convergence analyses of MOR methods with error estimates [5,25].

In this work we extend the strategy to general linear systems of DAEs. We show that a regularization via an ε-embedding is feasible using the Kronecker canonical form. However, this approach exhibits disadvantages in corresponding numerical methods from the point of view of stability as well as that of algebraic complexity, cf. [12]. Hence we apply the singular value decomposition to achieve an alternative regularization. In each approach we prove the pointwise convergence of the transfer functions. Moreover, it follows the uniform convergence on compact domains. The theoretical properties allow for using MOR techniques within two scenarios. Firstly, we can reduce the regularized system of ODEs for small parameter ε, which yields an approximation to the original system of DAEs. Secondly, a parametric model reduction is considered and the limit $\varepsilon \to 0$ results in an approach for DAEs, where the quality of the approximation still has to be investigated.

The paper is organized as follows. In Section 2, we briefly review the analysis of the input–output behavior of linear dynamical systems in frequency domain to apply MOR. The semi-explicit systems of DAEs and the regularization technique are introduced in Section 3. We extend this approach to general linear systems of DAEs in Section 4. Finally, the results of numerical simulations of two illustrative examples are presented.

2. Linear dynamical systems

A continuous time-invariant (lumped) multi-input multi-output linear dynamical system can be derived from an RLC circuit by applying modified nodal analysis (MNA),[1] see [16]. The system is of the form

$$\begin{cases} C\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= -Gx(t) + Bu(t) \\ w(t) &= Lx(t) + Du(t) \end{cases} \tag{1}$$

with initial condition $x(0) = x_0$. Here t is the time variable, $x(t) \in \mathbb{R}^n$ is referred as inner state (and the corresponding n-dimensional space is called state space), $u(t) \in \mathbb{R}^m$ is an input, $w(t) \in \mathbb{R}^p$ is an output. The dimensionality n of the state vector is called the order of the system. The number of inputs and outputs is m and p, respectively, and $C, G \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $L \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$ are the (constant in time) state space matrices. Without loss out generality we assume $D = 0$. Moreover, we assume that C, G, and B exhibit the block structure[2]:

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & G_2 \\ -G_2^T & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \tag{2}$$

where the sub-blocks C_1, G_1, and B_1 have the same number of rows, and

$$C_1 \geq 0, \; G_1 \geq 0, \; C_2 > 0. \tag{3}$$

Here, $\star \geq 0$ or $\star > 0$ means that $\star$ is symmetric and positive semi-definite or positive definite, respectively. The condition (3) implies that the matrices C and G satisfy:

$$G + G^T \geq 0 \text{ and } C \geq 0.$$

The matrices C and G in (1) are allowed to be singular, and we only assume that the pencil $G + sC$ is regular, i.e., the matrix $G + sC$ is singular only for a finite number of values $s \in \mathbb{C}$. For more details on existence and uniqueness of a solution, see [17]. Basically, MOR techniques aim to derive a system

$$\begin{cases} \tilde{C}\frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} &= -\tilde{G}\tilde{x}(t) + \tilde{B}u(t), \quad \tilde{x}(t) \in \mathbb{R}^q \\ \tilde{w}(t) &= \tilde{L}\tilde{x}(t) + \tilde{D}u(t), \quad \tilde{x}(0) = \tilde{x}_0, \quad \tilde{w}(t) \in \mathbb{R}^p, \end{cases} \tag{4}$$

of order q with $q \ll n$ that can replace the original high-order system (1) in the sense that the input–output behavior of both systems nearly agrees within some frequency band $\{s \in \mathbb{C} : |s| \leq S\}$. A common way is to identify a subspace of dimension $q \ll n$, that captures the dominant information of the dynamics and to project (1) onto this subspace, spanned by some basis vectors $\{v_1, \ldots, v_q\}$. The linear system of the form (1) is often referred to as the representation of the system in time domain, or in the state space. Equivalently, one can also represent the system in frequency domain via the Laplace transform. Recall that for a vector-valued function $f(t)$, the Laplace transform is defined component-wise by

[1] Note that our analysis and methods are not restricted to MNA systems, the method could be applied directly to the linear dynamical system in general form.

[2] If we apply the standard MNA [16].