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A volume integral equation method for periodic scattering problems for anisotropic Maxwell's equations

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ABSTRACT

This paper presents a volume integral equation method for an electromagnetic scattering problem for three-dimensional Maxwell's equations in the presence of a biperiodic, anisotropic, and possibly discontinuous dielectric scatterer. Such scattering problem can be reformulated as a strongly singular volume integral equation (i.e., integral operators that fail to be weakly singular). In this paper, we firstly prove that the strongly singular volume integral equation satisfies a Gårding-type estimate in standard Sobolev spaces. Secondly, we rigorously analyze a spectral Galerkin method for solving the scattering problem. This method relies on the periodization technique of Gennadi Vainikko that allows us to efficiently evaluate the periodized integral operators on trigonometric polynomials using the fast Fourier transform (FFT). The main advantage of the method is its simple implementation that avoids for instance the need to compute quasiperiodic Green's functions. We prove that the numerical solution of the spectral Galerkin method applied to the periodized integral equation converges quasioptimally to the solution of the scattering problem. Some numerical examples are provided for examining the performance of the method.

1. Introduction

Scattering of electromagnetic waves by periodic structures is not only an interesting mathematical topic but also of great interest in applications, e.g., for the construction and optimization of optical filters, lenses, and beam-splitters in optics. An overview about this and further topics in applied mathematics related to wave propagation in periodic structures can be found in, e.g., [3,31]. We consider in this paper scattering of time-harmonic electromagnetic waves from a three-dimensional biperiodic grating consisting of anisotropic dielectric materials. By biperiodic, we mean that the grating is periodic in the, say, x_1- and x_2-direction, while it is bounded in the x_3-direction. The scattering problem under consideration is described by time-harmonic anisotropic Maxwell's equations in three dimensions, completed by a physical radiation condition for the quasiperiodic scattered field. It is well known that efficient numerical simulation of the solution to this scattering problem is not an easy task because of typical difficulties of three-dimensional anisotropic Maxwell's equations. Further challenges may come from the quasiperiodicity, the radiation condition that one has to take into account, and from presence of evanescent waves around the biperiodic structure. Therefore, it might be difficult to use, e.g., a standard finite element software to simulate wave fields in such problems efficiently. For this reason, this paper firstly studies solution theory

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for the scattering problem using volume integral equations, and secondly presents a simple-to-implement solver for this simulation task.

Volume integral equations are a popular tool for numerically solving scattering problems in the engineering community, see, e.g., [4,6,11,22,28,34,40,41]. However, the discretization of the integral operator itself is sometimes done in a crude way, and a convergence analysis of the technique is often missing, in particular when material parameters are not globally smooth. Recently, volume integral equations also started to attract interest in the applied mathematics community. The papers [7,14–16,23,39] provide numerical analysis for the Lippmann–Schwinger integral equation, when the integral operator is compact. Further, [9,19,21,33] analyze strongly singular integral equations (i.e., integral operators that fail to be weakly singular) for scattering from bounded media. However, [33] considers media with globally continuous material properties, and the L^2-theory in [21] does not yield physical solutions if the material parameter appearing in the highest-order coefficients is not smooth. The paper [19] analyzes well-posedness of a strongly singular integral equation but it is not clear how its approach can be extended for studying numerical discretization. The authors in [9] prove a Gårding inequality for a strongly singular volume integral equation arising from electromagnetic scattering from a (discontinuous) dielectric body. This implies the convergence of Galerkin discretization. However, setting up the full system matrix is costly both in terms of memory and CPU time. The strong singularity of the integral kernel even makes the computation of the diagonal of the system matrix challenging. More recently, the papers [25,26], which are discussed in more details below, have studied both theoretical and numerical aspects for strongly singular volume integral equations for scattering problems by two-dimensional periodic structures. The present work can be considered as a generalization of the results in [25,26] to the case of periodic scattering for anisotropic Maxwell's equations.

Our first aim in this paper is to study well-posedness of the scattering problem by proving Gårding-type estimates for the equivalent volume integral equation. The most related result to this study is the paper [25]. As we mentioned, in [25], volume integral equations have been analyzed for scalar periodic scattering problems. The authors have first proved Gårding inequalities for strongly singular integral equations in weighted Sobolev spaces. Such inequalities have then been extended to standard Sobolev spaces, relying on isomorphism between the two spaces. This extension nevertheless requires that the periodic material is isotropic, and more regularity on the contrast and its support have to be assumed. The approach in [25], which crucially depends on the setting of the scalar problem, is no longer valid for the case of anisotropic Maxwell's equations. To treat the latter case we investigate Gårding-type estimates for the corresponding integral equation in a suitable truncation of the unit cell instead of the periodic support as in [25]. This, roughly speaking, enables the use of variational formulation of the integral operators which allows us to prove a desired Gårding-type estimate directly in standard Sobolev spaces $H(\mathrm{curl})$. This approach further improves the regularity assumption used in [25] on the material parameters and their periodic support. The result can be easily transferred to free space scattering problems.

Our second aim is to rigorously analyze a spectral Galerkin method to numerically solve the scattering problem, again for discontinuous media. We use the periodization technique of Gennadi Vainikko [39], where a corresponding collocation method for volume integral equations involving a compact integral operator has been analyzed. More precisely, this technique allows us to efficiently evaluate the periodized integral operators on trigonometric polynomials using the fast Fourier transform (FFT). Iterative methods can then be used for solving the periodized integral equation discretized by trigonometric collocation methods. For the case of strongly singular integral equations for periodic scattering in two dimensions, this method has been generalized as trigonometric Galerkin methods [26]. Central to the study of convergence analysis of such Galerkin methods is again Gårding-type estimates for the periodic integral equation. In the present paper, this is proved relying in part on the Gårding-type estimate for the original integral equation. Quasioptimal convergence of spectral Galerkin methods applied to the periodic integral equation is then established in terms of Céa's lemma. A first-order rate of convergence can further be observed from some numerical examples. We also give fully discrete formulas how to implement this method. Finally, we describe a couple of numerical examples.

It is also worth to mention that if the material parameters are piecewise constant then boundary integral equations turn out to be more efficient than the volume approach. We refer to [13,30] for a recent reference of boundary integral equation approaches dealing with periodic scattering problems. However, boundary integral equation methods usually need efficient and rapid evaluations of quasiperiodic Green's functions which is well known a non-trivial task, see, e.g., [27]. The numerical scheme we propose in this approach does not require to evaluate Green's functions and it is in principle applicable to arbitrary varying material parameters. Further, the method is simple to implement, and that the linear system can be evaluated at FFT speed. Of course, the convergence order is low if the medium has jumps, due to the use of global basis functions. However, if the material properties are globally smooth, then the method is high-order convergent. The technique is an interesting tool for numerical simulation, as we demonstrate through numerical examples.

The rest of paper is organized as follows: we provide in Section 2 a problem setting for the direct scattering problem. While Section 3 is dedicated to the volume integral equation formulation of the scattering problem, we prove in Section 4 a Gårding-type estimate for the strongly singular integral equation. In Section 5 we periodize the integral equation, and prove a Gårding-type estimate for the periodized integral equation. In Section 6, the periodized integral equation is discretized by spectral Galerkin methods whose convergence has been established thanks to the study in Section 5. We further give fully discrete formulas in this section. Section 7 contains numerical examples for examining the performance of the method.