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Effect of bulk viscosity in supersonic flow past spacecraft

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ABSTRACT

In this paper, we consider the effect of bulk viscosity in various hydrodynamic problems. We numerically study this effect on the front structure of the one-dimensional stationary shock wave and on the flow past blunt body. We estimate the effect of the bulk viscosity coefficient (BVC) on the heat transfer and drag of a sphere in a supersonic flow, apparently for the first time, by the numerical solution of parabolized Navier–Stokes equations. The solution is obtained by an original fast convergent method of global iterations of the longitudinal pressure gradient. The directions of further investigations of bulk viscosity are suggested.

1. Introduction

Presently, it is known, though not widely known, that the classical form of Navier–Stokes (NS) equations for a compressible gas can be applied, at best, to not very dense monoatomic gases only if the Stokes hypothesis is used [22], which relates the shear viscosity coefficient μ to the second viscosity coefficient μ' as:

$$2\mu + 3\mu' = 0, \tag{1.1}$$

which is equivalent to the assumption that the bulk viscosity coefficient (BVC) defined by relation (1.2) is zero:

$$\mu_b = \mu' + \frac{2}{3}\mu = 0. \tag{1.2}$$

In the phenomenological conclusion of NS equations the relation for the stress tensor $\hat{P}$ is represented as

$$\hat{P} = (-p + \mu_b \nabla \cdot \vec{v})\hat{G} + 2\mu\hat{e}^0, \quad \hat{e}^0 = \hat{e} - \frac{1}{3}\nabla \cdot \vec{v}\hat{G}, \tag{1.3}$$

here $\hat{G}$ is a metric tensor, p is pressure, $\hat{e}$ is a deformation rate tensor. The introduced tensor $\hat{e}^0$ has the zero trace: $\sum_{i=1}^{3} e_{ii}^0 = 0$ – it is the rate of the shear tensor, it is responsible for viscous stresses connected with the shear medium movements only (form change without volume change).

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In the general case, in the stress tensor it is necessary to consider two independent viscosity coefficients: shear μ and BVC-μ_b.

The shear viscosity coefficient μ is multiplied by the shear rate tensor $\hat{e}^0$, and accordingly the term $2\mu\hat{e}^0$ in (1.3) is responsible for stresses produced by the form change without the volume change. The BVC term $\mu_b \nabla \cdot \vec{v}\hat{G}$ is responsible for the shear stresses connected with the bulk change without the form change (bulk rates of medium deformation).

Relations (1.3) are also deduced in the kinetic theory of gases. The solution of the Boltzmann kinetic equation by the Chapman–Enskog method in the second approximation of a distribution function by the Knudsen number for a gas with internal degrees of freedom yields the relations [11,15,16,20,26]:

$$\hat{P} = (p_{rel} - p + \mu_b \nabla \cdot \vec{v})\hat{G} + 2\mu\hat{e}^0, \tag{1.4}$$

in which one more term $p_{rel}\hat{G}$ appears as compared with (1.3) – p_{rel} is the so-called relaxation pressure. The relaxation pressure occurs when part of internal degrees of freedom of molecules is close to local thermodynamic equilibrium [15,16,20] and the other part is non-equilibrium. This term is small as compared with the pressure one [26] and is usually neglected, in this work p_{rel} is also neglected.

Kinetic theory of gases explains the physical nature of BVC, viz. the coefficient takes into account the difference between transitional temperature and temperature of internal degrees of freedom (modes). These two temperatures equalize in finite time. When this time is short as compared with the time it takes for the macroparameters to change, i.e. the deviation from equilibrium is weak, we can introduce one equilibrium temperature and take into account the weak deviation from equilibrium by adding $\mu_b \nabla \cdot \vec{v}$ to the equilibrium pressure p. Averaging three normal pressures in (1.3), we get the arithmetic mean pressure: $\bar{p} = -\frac{1}{3}\sum_{i=1}^{3} p_{ii} = p - \mu_b \nabla \cdot \vec{v}$ different from the equilibrium pressure p.

It is interesting to note that in the elasticity theory we introduce, analogue to the coefficient μ_b, the modulus of bulk compression K: $3K = 3\lambda + 2\mu$ where λ, μ are Lame's elasticity constants.

Though the availability of BVC results from both phenomenological [18,19,22] and kinetic [11,15,16,20,26] conclusions of the relation for stress tensor (1.3), the Stokes hypothesis ($\mu_b = 0$) is applied even now not only to monoatomic but also to polyatomic gases, in particular, to air when solving aerodynamic problems by NS equations. This is partly due to the fact that in the last quarter of the last century the Euler equations for an ideal gas and boundary layer equations were used in which the terms with the BVC were not available at all or were extra-order ones (low-order terms).

When considering an incompressible fluid (gas) flow, BVC completely drops from the equations, because it enters the term $\mu_b \nabla \cdot \vec{u}$, and $\nabla \cdot \vec{u} = 0$ for incompressible flows.

In fluid and gas mechanics manuals, BVC is either neglected or its physical nature is not described clearly, most often on an example of the sound propagation problem [16].

It should be remarked that Stokes himself did not consider his hypothesis as a definition. In [22], he stressed that he studied the case of incompressible or weakly compressible medium, i.e. when the bulk viscosity is really negligible as $\nabla \cdot \vec{\upsilon} = 0$. Therefore, he assumed that $\mu_b = 0$, but he did not consider that the assumption is applicable to any flows as a hypothesis. Thus, we are to consider examples of problems, where the Stokes hypothesis cannot be used, but taking account of the bulk viscosity is of importance.

Evidently, Tisza was the first to try to take into account the bulk viscosity effect [25]. In problems of ultrasonic wave propagation in gases and fluids the theory predicts that the absorption coefficient is times less than the experimental values. Tisza supposed that the additional energy dissipation is due to the relaxation of internal degrees of freedom and tried to take account of this effect using the bulk viscosity coefficient. The value of the absorption coefficient predicted by theory without account taken of the bulk viscosity was defined by the formula $\alpha = \frac{2\pi^2 f^2}{\rho a^3}(\frac{4}{3}\mu + \frac{\gamma-1}{c_p}\lambda)$ where f is the frequency of sound waves, a is the speed of sound, λ is the heat conduction coefficient. With account taken of the bulk viscosity, one more term in brackets appeared: $\alpha = \frac{2\pi^2 f^2}{\rho a^3}(\frac{4}{3}\mu + \frac{\gamma-1}{c_p}\lambda + \mu_b)$.

Taking account of the bulk viscosity allowed one to eliminate the discrepancies with experiment. Thus, Tisza pointed to BVC importance. It should be noted that Tisza in his work also emphasized that taking account of BVC can be important for large compressibility problems and the introduction of μ_b is possible only if the local thermodynamic equilibrium condition is satisfied.

Thus, the Stoke relations hold only for the case of a monoatomic gas that is rarified so that triple and higher order collisions can be neglected but not to the extent to which the changes in macroscopic parameters of the mean free path of molecules can be neglected.

Emanuel et al. [8–10] showed that taking account of the bulk viscosity can strongly affect the flow characteristics. They studied two problems. The first problem – 2D Couette flow between two porous plates [10]. The lower plate is fixed, the upper one moves at a certain velocity. A liquid is injected through the lower porous plate at a certain velocity and sucked out through the upper plate at the same velocity. Taking account of the bulk viscosity significantly changes the density profile and twice changes the friction coefficient. Formulas for the bulk viscosity are not used, only the approximate value of $\mu_b = 2000\mu$, which is in agreement with the estimate of this coefficient for CO_2 at room temperature.

In [8], a hypersonic flow past a plane plate is calculated by the boundary value equations with account taken of the bulk viscosity (similar to $\mu_b = 2000\mu$). The calculations show that the heat flux increases by 50%, and the friction coefficient does not increase at all.