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A mesh simplification strategy for a spatial regression analysis over the cortical surface of the brain

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A B S T R A C T

We present a new mesh simplification technique developed for a statistical analysis of a large data set distributed on a generic complex surface, topologically equivalent to a sphere. In particular, we focus on an application to cortical surface thickness data. The aim of this approach is to produce a simplified mesh which does not distort the original data distribution so that the statistical estimates computed over the new mesh exhibit good inferential properties. To do this, we propose an iterative technique that, for each iteration, contracts the edge of the mesh with the lowest value of a cost function. This cost function takes into account both the geometry of the surface and the distribution of the data on it. After the data are associated with the simplified mesh, they are analyzed via a spatial regression model for non-planar domains. In particular, we resort to a penalized regression method that first conformally maps the simplified cortical surface mesh into a planar region. Then, existing planar spatial smoothing techniques are extended to non-planar domains by suitably including the flattening phase. The effectiveness of the entire process is numerically demonstrated via a simulation study and an application to cortical surface thickness data.

1. Introduction and motivation

In this paper, we develop a technique to analyze large data sets lying on complicated two-dimensional manifolds. In particular, we are interested in analyzing data observed over the cortical surface of the brain, a two-dimensional manifold with many folds and creases, constituting the outermost part of the brain. The data of interest are the hemodynamic signals associated with neural activity on the cerebral cortex, or the measurements of the cerebral cortex thickness (i.e., the thickness of grey matter tissue). From a medical viewpoint, the study of these data is of relative importance to better understand brain functions and the underlying mechanics of brain diseases. For instance, the thickness of the cerebral cortex changes over time and is linked, in the medical literature, to the pathology of many neurological disorders such as autism, Alzheimer's disease and schizophrenia [19]. Cortical surface data are obtained from reconstructions of the output of various types of magnetic resonance imaging (MRI) (see, e.g., [5]). Fig. 1 shows an example of thickness data studied in [4] and [3]. On the left, a cortical surface mesh is provided, while, on the right, we have the corresponding thickness measurements at each node of the mesh represented as a color map, obtained by linearly interpolating the measurements at the mesh nodes. Due to the folded nature of the cerebral cortex, the mesh generation process is a complex multistep procedure that results in a very large data set (often more than 10^6 nodes). Moreover, these data sets are usually characterized by noise in

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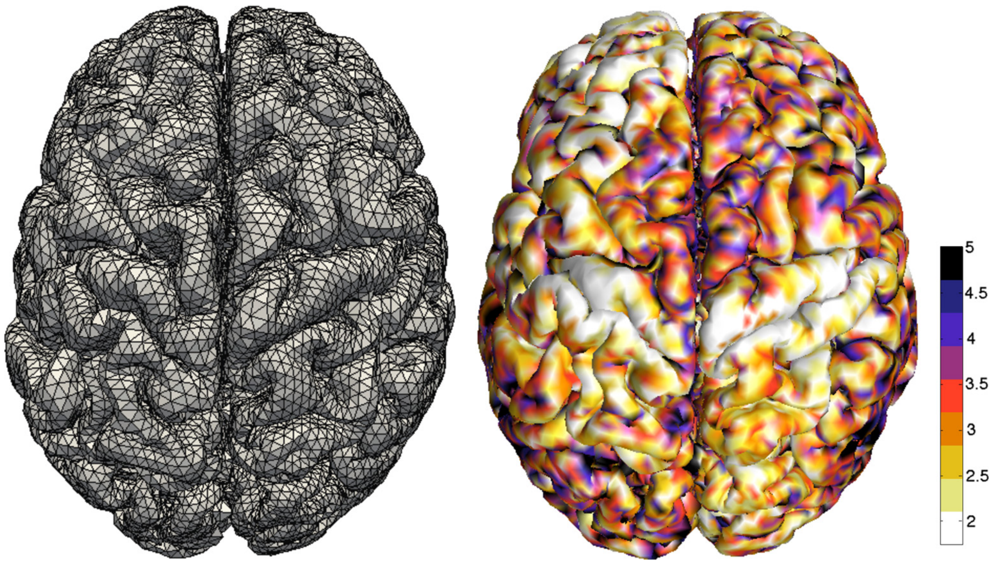

Fig. 1. Example of a cortical thickness data set: a cortical surface mesh with 40 962 nodes (left); color map of the cortical thickness (right). The data and the Matlab code used to build the color map are available at http://www.stat.wisc.edu/~mchung/softwares/hk/hk.html.

both the node locations and the data measurements. Advanced methods for modeling data spatially distributed over these manifolds are consequently required.

We propose an efficient technique to analyze large noisy data sets associated with triangular meshes of complicated non-planar geometries. To do this, we couple a mesh simplification technique with a spatial regression method for analyzing data on non-planar domains. The motivation for the simplification procedure is to reduce the computational effort associated with the statistical analysis of the large data sets that are typical in cortical surface applications. The proposed simplification procedure is designed specifically for producing a mesh that does not distort the original data distribution and is optimal for a statistical analysis of the data. In particular, through an iterative procedure, we take into account both the geometry of the mesh and the data distribution over it. The simplified geometry is generated in a way such that the analysis of the data associated with it should have statistical estimates with good inferential properties. For the data analysis, we resort to the Spatial Regression model for Non-Planar domains (SR-NP) developed in [9]. The SR-NP approach smooths the noisy data by minimizing a sum of squared error functional with a roughness penalty term involving the Laplace–Beltrami operator associated with the non-planar domain. The estimation problem on the surface is then appropriately recast over a planar domain via a conformal map. In the planar domain, existing spatial smoothing techniques are generalized by suitably taking into account the flattening of the domain. Notice that mapping to a planar domain would also allow for a statistical analysis across patients, similar to mapping to a reference brain [3]. In fact, via the SR-NP method, patient-specific estimates can all be mapped to a common planar domain where, after suitable registration among patients, comparisons across patients can be made. Nevertheless, the development of full inferential and uncertainty quantification tools for these population studies is outside the scope of this current paper. However, the mesh simplification proposed in this paper lays a foundation for these tools. The original application for the SR-NP method was modeling hemodynamic forces on the carotid artery (or on any manifold topologically equivalent to a cylinder). Since the cortical surface can be represented by a topological sphere, the conformal map has to be modified accordingly. To accomplish this, we implement a modified version of the conformal map suggested in [1]. The modification we introduce provides robust results when flattening some of the undesirable triangulations generated by the segmentation and extraction procedures [5].

Alternative approaches proposed in the literature chose different methods for containing the computational cost associated with the analysis of large cortical surface data sets. The nearest neighbor averaging technique developed in [12] is an iterative technique that smooths the variable of interest observed at each vertex of the mesh by suitably averaging this value with the ones observed at the neighboring vertices. The averaging process is repeated several times to create a smoothing effect. Although this technique is practical for smoothing data over the cortical surface, more sophisticated methods have been developed to build inferential tools that measure the uncertainty of the resulting estimates. For example, a recent method proposed in [19] identifies the mesh with a weighted graph. Then, the data associated with the mesh is smoothed by tuning the local support around each vertex of the graph via a graph Laplacian. Another example of a smoothing technique for neuroimaging applications is the Iterative Heat Kernel (IHK) smoothing introduced in [3]. This geodesic distance based kernel smoothing method solves the Laplace–Beltrami eigenvalue problem directly on the surface to construct a basis for the heat kernel on the cortical surface. Then, a finite number of these basis functions are used in the expansion of the heat kernel. In particular, a smoothing window is defined around each data point. The size of the smoothing window is