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Integral equations requiring small numbers of Krylov-subspace iterations for two-dimensional smooth penetrable scattering problems

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ABSTRACT

This paper presents a class of boundary integral equations for the solution of problems of electromagnetic and acoustic scattering by two-dimensional homogeneous penetrable scatterers with smooth boundaries. The new integral equations, which, as is established in this paper, are uniquely solvable Fredholm equations of the second kind, result from representations of fields as combinations of single and double layer potentials acting on appropriately chosen regularizing operators. As demonstrated in this text by means of a variety of numerical examples (that resulted from a high-order Nyström computational implementation of the new equations), these "regularized combined equations" can give rise to important reductions in computational costs, for a given accuracy, over those resulting from previous iterative boundary integral equation solvers for transmission problems.

1. Introduction

Owing mainly to the dimensional reduction and the absence of dispersion errors inherent in boundary integral formulations, scattering solvers based on boundary integral equations can be significantly more efficient than solvers resulting from direct discretization of the corresponding Partial Differential Equations (PDE). In view of the large dense systems of linear equations that result from integral equation formulations, however, integral-equation solvers often rely on use of Krylov subspace iterative solvers in conjunction with fast algorithms [6,9,36] for evaluation of matrix vector products. Since the numbers of iterations required by Krylov subspace solvers for a given system of equations depends greatly on the spectral character of the system matrix, the efficiency of boundary iterative integral solvers hinges on the spectral properties of the integral formulations used.

In this paper we introduce a new class of boundary integral equations for the solution of electromagnetic and acoustic transmission problems (that is, problems of diffraction and scattering by penetrable scatterers) for two-dimensional homogeneous penetrable scatterers with smooth boundaries. The new formulations, which, as established in this paper, are expressed in terms of uniquely solvable Fredholm equations of the second kind, result from representations of fields as combinations of single and double layer potentials acting on regularizing operators analogous to those used in [2,4,5,7,10–13,

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27–29] for non-penetrable scatterers. As demonstrated in this text by means of a variety of numerical examples, for a given accuracy, the new “regularized combined equations” can give rise to significant reductions in iteration numbers, and therefore, in overall computational costs, over those resulting from previous boundary integral formulations. Extensions of the techniques introduced in this paper to non-smooth boundaries and three dimensional problems is currently underway. We are also investigating the possibility of adapting the regularization procedure put forth in this paper to the important case of dielectric materials with junction points that has recently received significant attention in the literature [14,15,18,34].

Uniquely solvable formulations for transmission scattering problems have been available for quite some time [32]. For example, a class of such integral equations results from representation of the fields inside and outside the dielectric scatterer by means of Green’s formula: linear combinations of the interior and exterior boundary values of the fields and their normal derivatives can be used to produce various types of pairs of boundary integral equations with two unknown functions, including: (1) A family of integral equations *of the second kind* (which feature multiples of the identity operator and linear combinations of compact operators, including the single and double layer operators as well as suitable *compact* differences of hypersingular integral operators [19,24,32,35]); and (2) Integral equations *of the first kind with positive principal part* which include non-compact hypersingular operators. Equations of the type (1) and (2) are used most frequently as part of integral solvers for wave transmission problems.

(It is also possible to express the transmission problem in terms of a *single* uniquely solvable integral equation; the papers [20,26] present comprehensive discussions in these regards. It is useful to mention here that, while single transmission integral equations contain half as many unknowns as the corresponding systems, they require operator compositions which, in the context of iterative solvers considered in this paper, may lead to equal or even higher computing costs than the system formulations such as those mentioned above.)

This paper presents novel *regularized* integral equation formulations for the transmission problem. Unlike the second-kind formulations introduced earlier, which rely on cancellation of hypersingular terms in the integral kernels, the present approach produces second-kind formulations via *regularization of hypersingular operators by means of suitable regularizing operators* [10–13,28,29]. In particular, use of regularizing operators given by layer-potentials with *complex wavenumbers* results in second-kind Fredholm equations with improved spectral properties. Similar regularized equations result from use of regularizing operators expressed in terms of *Fourier operators* whose symbols have the same high-frequency asymptotics as the corresponding layer-potential regularizers mentioned above.

In order to demonstrate the properties of the new equations we introduce Nyström implementations of the previous and new formulations considered. These implementations, which are based on the methods introduced in [8,22,25,31], include elements such as approximation via global trigonometric polynomials, splitting of integral kernels into singular and smooth components, and explicit quadrature of products of logarithmically singular terms and trigonometric polynomials.

This paper is organized as follows: in Sections 2, 3, and 4 we review the classical systems of integral equations for the solution of transmission scattering problems, we introduce the new regularized formulations, and we establish their unique solvability and second-kind Fredholm character in the appropriate functional spaces. The reductions in iteration numbers and computing times that result from the new formulations are demonstrated in Section 5.

2. Acoustic transmission problems

We consider the problem of evaluation of the time-harmonic fields that arise as an incident field u^{inc} impinges upon a homogeneous dielectric scatterer which occupies a bounded region $\Omega_2 \subset \mathbb{R}^2$. Calling $\Omega_1 = \mathbb{R}^2 \setminus \Omega_2$ the region exterior to the obstacle, in what follows ϵ_1 and ϵ_2 denote the electric permeabilities of the materials in regions Ω_1 and Ω_2, respectively; the permeabilities of both dielectrics is assumed to equal μ_0, the permeability of vacuum. Letting u^1 and u^2 denote the scattered field in Ω_1 and the total field in Ω_2, respectively, calling Γ the boundary between the domains Ω_1 and Ω_2 (which, throughout this text is assumed to be smooth), and given an incident field u^{inc} for which

$$\Delta u^{inc} + k_1^2 u^{inc} = 0 \quad \text{in } \mathbb{R}^2, \tag{1}$$

the fields u^1 and u^2 can be obtained as the solutions to the Helmholtz equations

$$\Delta u^1 + k_1^2 u^1 = 0 \quad \text{in } \Omega_1, \tag{2}$$

$$\Delta u^2 + k_2^2 u^2 = 0 \quad \text{in } \Omega_2, \tag{3}$$

which satisfy the transmission conditions

$$\begin{aligned} & u^1 + u^{inc} = u^2 \quad \text{on } \Gamma, \\ & \frac{\partial u^1}{\partial n} + \frac{\partial u^{inc}}{\partial n} = \nu \frac{\partial u^2}{\partial n} \quad \text{on } \Gamma \end{aligned} \tag{4}$$

as well as the Sommerfeld radiation condition at infinity:

$$\lim_{|r| \to \infty} r^{1/2} \left(\partial u^1 / \partial r - i k_1 u^1 \right) = 0. \tag{5}$$