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A predictor–corrector approach for pricing American options under the finite moment log-stable model

Wenting Chen [a], Xiang Xu [b,*], Song-Ping Zhu [c,d]

[a] School of Mathematics and Applied Statistics, University of Wollongong, NSW 2522, Australia
[b] Department of Mathematics, Zhejiang University, Zhejiang, 310058, China
[c] School of Mathematics and Applied Statistics, University of Wollongong, NSW 2522, Australia
[d] JiLin University, JiLin, 130012, China

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A B S T R A C T

This paper investigates the pricing of American options under the finite moment log-stable (FMLS) model. Under the FMLS model, the price of American-style options is governed by a highly nonlinear fractional partial differential equation (FPDE) system, which is much more complicated to solve than the corresponding Black–Scholes (B–S) system, with difficulties arising from the semi-globalness of the fractional operator, in conjunction with the nonlinearity associated with the early exercise nature of American-style options. Albeit difficult, in this paper, we propose a new predictor–corrector scheme based on the spectral-collocation method to solve for the prices of American options under the FMLS model. In the current approach, the nonlinearity of the pricing system is successfully dealt with using the predictor–corrector framework, whereas the non-localness of the fractional operator is elegantly handled. We have also provided an elegant error analysis for the current approach. Various numerical experiments suggest that the current method is fast and efficient, and can be easily extended to price American-style options under other fractional diffusion models. Based on the numerical results, we have also examined quantitatively the influence of the tail index on American put options.

1. Introduction

As is well known, one of the major topics in today's quantitative finance research is the valuation of option derivatives. For quite a long time, it has been widely acknowledged that pricing American options is a much more intriguing problem [12,13,15], with challenge mainly stemming from the nonlinearity originated from the inherent characteristic that an American option can be exercised at any time during its lifespan. This additional right of being able to exercise the option early, in comparison with a European option, casts the American option pricing problem into a free boundary problem, which is highly nonlinear and far more difficult to deal with. Since most traded stock and commodity options in today's financial markets are of American style, it is important to ensure that American-style securities can be priced accurately as well as efficiently.

In the literature, the pricing of American options has been well documented, but most of them are discussed under the commonly used Black–Scholes (B–S) model. Empirical evidence has, however, suggested that the B–S model usually

* Corresponding author.
E-mail addresses: xxu@zju.edu.cn (X. Xu), spz@uow.edu.au (S.-P. Zhu).

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underestimates the probability of underlying price moving significantly over small time steps [3]. For example, when analyzing the S&P 500 data, a "leptokurtic distribution" is observed, which has a higher peak and two heavier tails than those of the normal distribution [3]. To incorporate the leptokurtic feature observed in financial markets, the Lévy process is introduced, which is a vast class including the standard Brownian motion, Poisson process and compound Poisson processes as its simplest forms. In addition to the fat tails they have, Lévy distributions allow for long jumps as well, which are frequently observed in stock markets [3].

Among all the Lévy processes, the maximally skewed Lévy stable (LS) process introduced in [3] has received a great attention, due to its financial and mathematical superiorities. This special Lévy process gives rise to an interesting financial model known as the finite moment log-stable (FMLS) model [3], which is also adopted in the current work. Empirical evidence suggests that this model can not only successfully capture the high-frequency empirical probability distribution of the S&P 500 data, but also fit simultaneously volatility smirks at different maturities [3]. Most importantly, in contrast to many other models driven by different Lévy processes, the FMLS model guarantees that all moments of the underlying index levels are finite, which are needed for the existence of an equivalent martingale measure and finite option values [3].

Mathematically, to characterize the non-localness induced by the pure jumps under the FMLS model, a fractional partial differential equation (FPDE) needs to be solved, which is a subset of the class of pseudo-differential equations. We remark that this new FPDE system is similar to the corresponding B–S system, but with the second-order spatial derivative replaced by an α-order spatial derivative, where α is any real number belonging to $(1, 2]$. The non-local nature of the factional operator in fact weights information of the portfolio over a range of underlying values rather than narrowly focusing on some localized information. Financially, this reflects the fact that over a time step Δt, the underlying price S_t can diffuse and/or jump to value $S_{t+\Delta t}$ far away from S_t, making the use of localized information of the portfolio at S_t less relevant.

Albeit important, the application of fractional calculus to the quantitative finance area is very much in its infancy stage, due to the difficulty in dealing with the non-local nature of the fractional operator. For instance, Cartea [4] showed that hedging strategies can be substantially improved once fractional operators are adopted. Wyss [22] considered the pricing of option derivatives under a modified B–S equation with a time-fractional derivative, and derived a closed-form solution for European vanilla options. Regarding the option pricing under the FMLS model, Carr and Wu [3] introduced this model to the literature, and showed its superior performance against several widely used alternatives. Cartea and del-Castillo-Negrete [5] considered the pricing of barrier options under the FMLS model purely numerically, by using a finite difference method. Recently, Chen et al. [9] derived closed-form analytical solutions for European-style options under the same model, and analyzed quantitatively the impact of different tail indexes. It should be remarked that under the fractional diffusion framework, the literature has been restricted to the pricing of European-style options only. The pricing of American-style options, which is the main concern of today's financial industry, is much more complicated than the one under the B–S model, with the additional difficulty arising from the non-localness of the fractional operator. To the best of the authors' knowledge, this important issue has only been considered recently in [7,8], although quite a lot of work has been done for American options under normal diffusion processes [26–28].

In this paper, an efficient numerical scheme for the pricing of American options under the FMLS model is proposed. Our approach is very much based on the predictor–corrector framework, which is commonly used to numerically solve nonlinear PDEs. The idea behind the predictor–corrector method is to use a suitable combination of an explicit and implicit technique to obtain a method with better convergence characteristics. Previously, this method has been applied to the pricing of American options under the normal diffusion frameworks only, such as the B–S model [28], and the stochastic volatility model [26]. The purpose of this article is to introduce a new predictor–corrector scheme based on the spectral-collocation method, which is suitable for not only the FMLS model, but also various models driven by different fractional diffusion processes. Due to the highly nonlinear nature of the pricing of American options, it is never an easy task to do any numerical analysis for the current approach. Nevertheless, by using error splitting, we still have managed to provide an elegant error analysis for our predictor–corrector approach. It is found that the convergence rate depends on the regularity of the solution, which is the nature of the spectral-collocation method.

In the subsequent sections, we shall present this new approach together with several numerical results for American puts under the FMLS model. The paper is organized as follows: In Section 2, we introduce the FPDE system that the price of an American put must satisfy under the FMLS model. In Section 3, we present our predictor–corrector approach in detail. In Section 4, the error analysis for the current method is provided. In Section 5, numerical examples together with some useful discussions are presented to demonstrate the convergence, efficiency and accuracy of the current scheme. Concluding remarks are given in the last subsection.

2. American options under the FMLS model

Under the risk neutral measure $\mathbb{Q}$, the FMLS model assumes that the log value of the underlying i.e., $\bar{x}_t = \ln S_t$, with dividend yield D follows a stochastic differential equation of the maximally skewed LS process:

$$d\bar{x}_t = (r - D - \nu)dt + \sigma dL_t^{\alpha,-1}, \tag{2.1}$$

where r and D are the risk free interest and the dividend yield, respectively. t is the current time, and $\nu = \sigma^\alpha \sec \dfrac{\alpha\pi}{2}$ is a convexity adjustment. $L_t^{\alpha,-1}$ denotes the maximally skewed LS process, which is a special case of the Lévy-α-stable