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Stability of an implicit method to evaluate option prices under local volatility with jumps

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ABSTRACT

In this paper, we consider a local volatility model with jumps under which the price of a European option can be derived by a partial integro-differential equation (PIDE) with nonconstant coefficients. In order to solve numerically the PIDE, we generalize *the implicit method with three time levels* which is constructed to avoid iteration at each time step. We show that the implicit method has the stability with respect to the discrete ℓ^2-norm by using an energy method. We combine the implicit method with an operator splitting method to solve a linear complementarity problem (LCP) with nonconstant coefficients that describes the price of an American option. Finally we conduct some numerical simulations to verify the analysis of the method. The proposed method leads to a tridiagonal linear system at each time step and thus the option prices can be computed in a few seconds on a computer.

1. Introduction

Derivative pricing problems can be formulated by the Black–Scholes equation if an underlying asset follows a geometric Brownian motion or its generalized Black–Scholes equation if an underlying asset in addition admits deterministic volatility function of underlying asset and time. During the last three decades, there were many evidences that these assumptions for the underlying asset cannot account for some features (e.g. volatility smiles in the former or co-movement of smiles and skews with the underlying in the latter) which are observed in many financial markets. In this paper, we consider a local volatility model with jumps [2] that can explain these phenomena under which option pricing problems can be derived by a partial integro-differential equation (PIDE) with non-constant coefficients

$$\frac{\partial u}{\partial \tau}(\tau,x) - \mathcal{L}u(\tau,x) = 0 \tag{1}$$

for all $(\tau,x) \in (0,T] \times (-\infty,\infty)$, where $\mathcal{L}u$ is the integro-differential operator given by

$$\mathcal{L}u(\tau,x) = \frac{\sigma(\tau,x)^2}{2}\frac{\partial^2 u}{\partial x^2}(\tau,x) + \left(r - d - \frac{\sigma(\tau,x)^2}{2} - \lambda\zeta\right)\frac{\partial u}{\partial x}(\tau,x)$$

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$$- (r+\lambda)u(\tau,x) + \lambda \int_{-\infty}^{\infty} u(\tau,z)f(z-x)dz. \tag{2}$$

and the volatility $\sigma(\tau,x)$ in (2) is a variable function with respect to the time and spatial variables.

In the case of jump-diffusion models when the volatility in (2) has constant coefficients (i.e. $\sigma(\tau,x) := \sigma$ for all τ, x), d'Halluin, Forsyth, and Vetzal [5] suggested an implicit method of the Crank–Nicolson type for evaluating European options in order to solve numerically the PIDE with constant coefficients. This implicit method has the second-order convergence rate in time and spatial variables. But, they applied a fixed point iteration to solve the linear system involving the inverse of a dense matrix at each time step. Kwon and Lee [8] developed a finite difference method, called the implicit method with three time levels, to evaluate the prices of the European options under jump-diffusion models with constant coefficients. To overcome the fixed point iteration at each time step, they used a numerical method with three time levels, which is shown to be stable in the sense of the Von Neumann analysis and to have the second-order accuracy in time and spatial variables when the PIDE has constant coefficients.

The objective of this paper is to generalize the implicit method proposed by Kwon and Lee [8] to the case of the PIDE with variable coefficients (1) and to show that the implicit method has the stability in the discrete ℓ^2-norm by using the energy method. Moreover, we propose the implicit method combined with the operator splitting method in [6,9] to solve the LCP with variable coefficients for evaluating American options. The numerical method is designed to avoid the fixed point iteration at each time step and we focus on the formulation of a tridiagonal linear system with variable coefficients. Thus it can be solved directly by using LU decomposition.

The remainder of this paper is organized as follows. In Section 2 we extend the implicit method with three time levels, which has been studied recently by Kwon and Lee [8], to the case of the PIDE with variable coefficients. In Section 3 it is proved that the implicit method is stable with respect to the discrete ℓ^2-norm. In Section 4 we combine the implicit method with an operator splitting method to solve the LCP with variable coefficients which describes the prices of the American options under jump-diffusion models. In Section 5 we perform some numerical simulations for pricing European and American put options under the Merton and Kou models to verify the analysis of the proposed method. The paper ends with conclusions in Section 6.

2. Discretization

In this section we introduce a finite difference method to discretize the PIDE having variable coefficients. Let us consider the following PIDE with initial and boundary conditions

$$\frac{\partial \bar{u}}{\partial \tau}(\tau,x) = \mathcal{L}\bar{u}(\tau,x), \quad (\tau,x) \in (0,T] \times (-X,X), \tag{3}$$

$$\bar{u}(\tau,x) = \bar{g}(\tau,x), \quad x \in \mathbb{R} \setminus (-X,X), \tag{4}$$

$$\bar{u}(0,x) = \bar{h}(x), \quad x \in (-X,X), \tag{5}$$

where $\mathcal{L}\bar{u}$ is the integro-differential operator defined by

$$\mathcal{L}\bar{u}(\tau,x) = \frac{(\sigma(\tau,x))^2}{2}\frac{\partial^2 \bar{u}}{\partial x^2}(\tau,x) + \alpha(\tau,x)\frac{\partial \bar{u}}{\partial x}(\tau,x) + \beta(\tau,x)\bar{u}(\tau,x) + \lambda \int_{-\infty}^{\infty} \bar{u}(\tau,z)f(z-x)dz,$$

$\tau = T - t$ is the remaining time to maturity T, $x = \ln(S/S_0)$ is the log price of the underlying asset S for a given price S_0, X is a positive constant, $\lambda > 0$ is the intensity of the jumps, and $f(x)$ is the density function of jump sizes of the log return x. The functions $\bar{g}(\tau,x)$ and $\bar{h}(x)$ are boundary and initial conditions, respectively.

We impose some assumptions, which are described in [1], on the variable coefficients in the initial and boundary valued problem (3)–(5) before we proceed to discretize the PIDE.

(A1) All variable coefficients σ, α, and β are continuous and sufficiently regular for the analysis of the proposed method.
(A2) There are $\underline{\sigma} > 0$ and $\bar{\sigma} > 0$ such that for all $(\tau,x) \in (0,T] \times [-X,X]$

$$0 < \underline{\sigma} < \sigma(\tau,x) < \bar{\sigma}.$$

(A3) There is $C_\sigma > 0$ such that for all $(\tau,x) \in (0,T] \times [-X,X]$

$$\left|\frac{\partial \sigma}{\partial x}(\tau,x)\right| < C_\sigma.$$