Applied Numerical Mathematics 87 (2015) 31–37

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

Choice of strategies for extrapolation with symmetrization in the constant stepsize setting

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ARTICLE INFO

Article history:
Received 16 June 2014
Received in revised form 15 August 2014
Accepted 20 August 2014
Available online 27 August 2014

Keywords:
Extrapolation
Symmetrizers
Symmetrization
Gauss methods
Lobatto IIIA methods and stiff problems

ABSTRACT

Symmetrization has been shown to be efficient in solving stiff problems. In the constant stepsize setting, we study four ways of applying extrapolation with symmetrization. We observe that for stiff linear problems the symmetrized Gauss methods are more efficient than the symmetrized Lobatto IIIA methods of the same order. However, for two-dimensional nonlinear problems, the symmetrized 4-stage Lobatto IIIA method is more efficient. In all cases, we observe numerically that passive symmetrization with passive extrapolation is more efficient than active symmetrization with active extrapolation.

1. Symmetrization

The symmetrizer constructed by Chan [1] has shown to be very efficient in solving stiff linear and nonlinear problems [4,2]. The extended smoothing by William Gragg [6] known as symmetrization is constructed so that it can preserve the asymptotic error expansions as well as to provide damping when solving stiff problems. The symmetrized 2-stage Gauss and symmetrized 3-stage Gauss methods are proven to suppress order reduction at least for linear stiff problems [2]. For example, the 2-stage Gauss method gives order 2 when solving stiff linear problems and with symmetrization, it yields order 4. Similar result is obtained for the 3-stage and 4-stage Lobatto IIIA methods in solving stiff linear problems. The theoritical analysis is given in [4]. However, the opposite is observed when solving stiff nonlinear problems. The symmetrized 3-stage Lobatto IIIA and symmetrized 4-stage methods although they do not suppress the order conditions, they are found to be very efficient in solving stiff nonlinear problems [4,5]. The summary of order for Gauss and Lobatto IIIA methods with passive and active symmetrization for linear and nonlinear scalar and matrix problems is given in Table 3. The construction of the symmetrized Gauss methods of order-3 ($\widetilde{\mathscr{G}}_2$), order-5 ($\widetilde{\mathscr{G}}_3$) and the symmetrized Lobatto IIIA methods which are also of order-3 ($\widetilde{\mathscr{L}}_3$) and order-5 ($\widetilde{\mathscr{L}}_4$) is given in [4]. Although the order of these symmetrized methods is less than the order of the base methods such as the 2-stage Gauss $\mathscr{G}_2$, 3-stage Gauss $\mathscr{G}_3$, 3-stage Lobatto IIIA $\mathscr{L}_3$ and the 4-stage Lobatto IIIA methods $\mathscr{L}_4$, in the passive mode we have observed that the order of the symmetrized methods is similar to the base methods. This result is true when observed numerically and analytically [4] because in the passive mode, symmetrizer is applied locally at the end of many steps and thus the global error gives $O(h^{p+1})$.

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http://dx.doi.org/10.1016/j.apnum.2014.08.004
0168-9274/

In our latest article [2], we have defined two ways of applying symmetrization (passive and active modes) without extrapolation. In this paper, we wish to investigate six different ways of implementing extrapolation with symmetrization. The choice of applying symmetrization with extrapolation is given in Section 2. In Section 3 we present and discuss the results of our numerical experiments in constant stepsize setting and the final section conclude the findings.

2. Choice of strategies for extrapolation

Extrapolation can be applied in two different forms; local (active) extrapolation [3] and global (passive) [8] extrapolation. In the active mode, the extrapolated value is used to propagate the numerical solution at the next step, while in the passive mode extrapolation is carried out only at points where greater accuracy is desired and where the unextrapolated solutions are propagated. Similarly, symmetrization can also be applied in active and passive modes. In an active mode, the symmetrized value is propagated whenever it is computed while in the passive mode the symmetrized value is not propagated [2].

To apply extrapolation, the base symmetric method should have an asymptotic error expansion in even powers of h so that we can obtain an increment of the order by two at a time. Since in the linear case, symmetrization can restore the classical order of the Gauss and Lobatto IIIA methods, we can therefore apply extrapolation with the correct formula to eliminate the leading error term and hence increase the accuracy.

With symmetrization, extrapolation can also be carried out in two modes; active and passive. It is of interest to know which mode is more efficient with active and passive symmetrization.

In the constant stepsize setting, there are six possible ways of applying extrapolation with symmetrization. These strategies are illustrated in Figs. 1–6. Extrapolation is carried out using stepsizes h and $h/2$. Here, we are considering only the first level of extrapolation.

The legend for all the figures is given in Table 1.

1. **No symmetrization with passive extrapolation**
   Fig. 1 shows passive extrapolation at x_3 and x_n of the symmetric solutions. The solid lines represent applying the symmetric solutions of the base method until the desired time x_3 and x_n while the vertical dashed lines show the linear combinations of two solutions with stepsizes h and $h/2$. Extrapolation is carried out whenever greater accuracy is required without propagating the extrapolated solutions.
2. **No symmetrization with active extrapolation**
   Fig. 2 shows active extrapolation at x_3 and x_n of the symmetric solutions. The solid lines represent the symmetric solutions of the base method while the vertical dashed lines represent the linear combinations of two solutions with stepsizes h and $h/2$. Extrapolation is carried out at each step and the extrapolated values are used to propagate the next solution.
3. **Passive symmetrization with passive extrapolation**
   Fig. 3 is about applying passive extrapolation with passive symmetrization at x_2. The solid lines represent the symmetric solutions of the base method and the gray vertical lines are the symmetrized solutions. The vertical dashed lines represent the linear combinations of two symmetrized values with stepsizes h and $h/2$. Extrapolation is carried out using the symmetrized values whenever greater accuracy is required without propagating the symmetrized extrapolated solution.
4. **Passive symmetrization with active extrapolation**
   Applying active extrapolation with passive symmetrization at x_2 is given in Fig. 4. Passive symmetrization is performed using the symmetric values $y_h(x_2)$ and $y_{\frac{h}{2}}(x_2)$ with stepsizes h and $h/2$. The gray circles are the symmetrized solutions. Extrapolation is then carried out using the symmetrized solutions with stepsizes h and $h/2$ and the symmetrized extrapolated solution is propagated.
5. **Active symmetrization with passive extrapolation**
   Fig. 5 shows passive extrapolation at x_2 with active symmetrization. The solid lines represent active symmetrization at every step while the vertical dashed lines represent the linear combinations of two symmetrized values with stepsizes h and $h/2$. Extrapolation is carried out whenever greater accuracy is required without propagating the symmetrized extrapolated solution.
6. **Active symmetrization with active extrapolation**
   Fig. 6 shows active extrapolation at x_2 with active symmetrization. The solid lines represent the symmetrized solutions while the vertical dashed lines denote applying active extrapolation using the two symmetrized values with stepsizes h and $h/2$. The symmetrized extrapolated value is used to propagate the solution.

In the variable stepsize setting, there are only two ways of applying symmetrization which are (4) and (6) given in Figs. 4 and 6 respectively. For (4), we stored the values of the base methods and the symmetrized solutions. We then propagate using the base values. The symmetrized values are stored to produce the outputs. Meanwhile for (6), the order of computations are similar with (4) except now we propagate with the symmetrized value and these values are stored as the outputs. Detailed work on symmetrization in the variable stepsize setting will be explained in another paper.