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An efficient semi-implicit finite volume method for axially symmetric compressible flows in compliant tubes

Michael Dumbser[a,*], Uwe Iben[b], Matteo Ioriatti[a]

[a] Laboratory of Applied Mathematics, Department of Civil, Environmental and Mechanical Engineering, University of Trento, Via Mesiano 77, 38123 Trento, TN, Italy
[b] Robert Bosch GmbH, Postfach 10 60 50, D-70049 Stuttgart, Germany

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ABSTRACT

In the present paper a new efficient semi-implicit finite volume method for the simulation of weakly compressible, axially symmetric flows in compliant tubes is presented. The fluid is assumed to be barotropic and a simple cavitation model is also included in the equation of state in order to model phase transition when the fluid pressure drops below the vapor pressure. The discretized flow equations lead to a mildly nonlinear system of equations that is efficiently solved with a nested Newton technique. The new numerical method has to obey only a mild CFL condition based on the flow velocity and not on the sound speed, leading to large time steps that can be used. The scheme behaves well in the presence of shock waves and phase transition, as well as in the incompressible limit. In the present approach, the radial velocity profiles and therefore the wall friction coefficient are directly computed from first principles. In the compressible regime, the new method is carefully validated against quasi-exact solutions of the Riemann problem, while it is validated against the exact solution found by Womersley for an oscillatory flow in a rigid tube in the incompressible regime.

1. Introduction

Hydraulic systems are used to transfer energy stored under the form of pressure into mechanical work. They consist of a large number of single components, such as pumps, valves and throttles and form a complex mechanical system. The components are often connected with elastic tubes. On the basis of pressure changes in the system the tube lines stretch. The stretch leads to a change of hydraulic system properties such as local speed of sound, eigen-frequencies or the time-dependent mass flow. The close interactions of fluid flow and tube strain cannot be neglected for highly transient or oscillatory flows. In order to forecast system properties, the fundamental relationship of pressure change and tube stretching has to be known. A complete three-dimensional modeling of the flow field in combination with the compliant tube structure leads to complex mathematical models and very time-consuming algorithms. If the flow field is assumed to be laminar and axially symmetric, the resulting one- or two-dimensional models are reasonably easy to handle and solvable in short computational times. It is therefore the aim of this paper to present an efficient algorithm for such reduced one- and two-dimensional models for axially symmetric flows in compliant tubes.

* Corresponding author.
E-mail addresses: michael.dumbser@unitn.it (M. Dumbser), uwe.iben@de.bosch.com (U. Iben), matteo.ioriatti@unitn.it (M. Ioriatti).

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While explicit density-based Godunov-type finite volume methods are usually very efficient and accurate in the case of strong shock waves and highly compressible flows, they are computationally inefficient in the low Mach number limit due to the severe time step restriction associated with the CFL condition for explicit schemes at low Mach numbers. They are furthermore not able to retrieve the elliptic behavior of the governing PDE in the incompressible limit. Many papers have studied the Riemann problem and finite volume methods for the compressible Euler equations and compressible multiphase flows in *rigid* ducts with spatially variable cross section, see for example [31,39,26,18,2,27,28,5], but much less is known about compressible gas flow in *elastic tubes* with variable cross section. A very nice comparison between the solution of the Riemann problem of reduced non-conservative one-dimensional flow models and the fully three-dimensional compressible Euler equations in a duct with axial symmetry can be found in [35].

The aim of the present paper is to devise a new efficient semi-implicit *pressure-based* finite volume scheme that is at the same time able to reproduce the features of compressible flows and which also works properly in the incompressible limit, without imposing a CFL condition based on the sound speed in the low Mach number regime. The present method is an extension of a family of semi-implicit schemes recently proposed for the simulation of incompressible blood flow in compliant arteries, see [14,38,24]. We also emphasize the direct relation of the present scheme to the method presented in [15] for the simulation of mixed free surface flows and pressurized flows of an incompressible fluid in pipes of an urban drainage system.

The outline of the paper is as follows: in Section 2 we first present the method for the one-dimensional inviscid case and then extend it to the 2D axially symmetric case, where also viscous effects are included and radial velocity profiles are directly obtained from first principles. In Section 3 we present a quasi-exact Riemann solver for inviscid weakly compressible flows in elastic tubes. In Section 4 the method is carefully validated against the quasi-exact solutions of the Riemann problem and against the exact solution of Womersley [41].

2. Numerical method

2.1. One-dimensional model

The compressible Euler equations for a weakly compressible inviscid barotropic fluid in an elastic duct with variable cross section read

$$\begin{aligned} &\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A u) = 0, \\ &\frac{\partial}{\partial t}(\rho A u) + \frac{\partial}{\partial x}\left(\rho A u^2\right) + A\frac{\partial p}{\partial x} = 0, \end{aligned} \tag{1}$$

where $x \in \Omega = [x_L, x_R]$ is the axial direction, ρ denotes the fluid density, A is the cross-sectional area, p is the fluid pressure and u is the average flow velocity in axial direction. The one-dimensional computational domain is denoted by Ω. The PDE system (1) is closed with an equation of state (EOS) for the pressure $p = p(\rho)$ and by an equation $A = A(p)$ that relates the cross sectional area to the pressure. The fluid is supposed to be barotropic, hence $p = p(\rho)$ and no energy equation is needed. In particular, we use the following relations: The tube area is given by the so-called law of Laplace [36,14], which is a simple elastic ring-model for the compliant behavior of the tube wall and is given by

$$A(p) = \pi R(p)^2, \quad \text{with } R(p) = \max\left(0, R_0 + (p - p_0)/\beta\right), \tag{2}$$

where R_0 is the equilibrium radius of the tube, p_0 is a given equilibrium pressure and β is a rigidity coefficient that can be linked to the wall thickness h_0, the Young modulus E and the Poisson ratio ν_P of the tube wall (see [36,24]) as

$$\beta = \frac{h_0 E}{(1-\nu_P)^2 R_0^2}. \tag{3}$$

For the equation of state of the barotropic fluid we use the EOS of a weakly compressible liquid together with a simple cavitation model given by the assumptions of local thermodynamic equilibrium and a homogeneous mixture of the liquid phase and the vapor phase:

$$\rho(p) = \begin{cases} \rho_0 + \frac{p - p_v}{c_0^2} & \text{if } p \ge p_v, \\ \frac{1}{\frac{\mu(p)}{\rho_v} + \frac{1-\mu(p)}{\rho_0}} & \text{if } 0 < p \le p_v, \end{cases} \tag{4}$$

where p_v indicates the vapor pressure, ρ_0 and c_0 are the reference mass density and the sound speed in the liquid, respectively, and $\mu(p) = -K(p - p_v)$ denotes the vapor mass fraction. For a more detailed discussion and applications of the homogeneous mixture approach to compressible cavitating flows see also [30,42,4,3,29]. The vapor is modeled by an ideal gas equation of state as $\rho_v = p/(R_v T_0)$, where T_0 is a reference temperature and R_v is the vapor gas constant. The quantity K is related to basic thermodynamic derivatives and is assumed to be constant in the following. For an efficient high order Godunov-type finite volume scheme able to handle complex tabulated equations of state with phase change