Applied Numerical Mathematics 83 (2014) 22–37

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

A variational approach to reconstruction of an initial tsunami source perturbation

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ARTICLE INFO

Article history:
Received 1 May 2013
Received in revised form 2 April 2014
Accepted 7 April 2014
Available online 19 April 2014

Keywords:
Reconstruction of initial tsunami source
Shallow water equations
Cost functional
Conjugate gradient algorithm
Numerical calculations

ABSTRACT

Tsunamis are gravitational, i.e. gravity-controlled waves generated by a given motion of the bottom. There are different natural phenomena, such as submarine slumps, slides, volcanic explosions, earthquakes, etc. that can lead to a tsunami. This paper deals with the case where the tsunami source is an earthquake. The mathematical model studied here is based on shallow water theory, which is used extensively in tsunami modeling. The inverse problem consists of determining an unknown initial tsunami source $q(x, y)$ by using measurements $f_m(t)$ of the height of a passing tsunami wave at a finite number of given points (x_m, y_m), $m = 1, 2, \dots, M$, of the coastal area. The proposed approach is based on the weak solution theory for hyperbolic PDEs and adjoint problem method for minimization of the corresponding cost functional. The adjoint problem is defined to obtain an explicit gradient formula for the cost functional $J(q) = \|Aq - F\|^2$, $F = (f_1, \dots, f_M)$. Numerical algorithms are proposed for the direct as well as adjoint problems. Conjugate gradient algorithm based on explicit gradient formula is used for numerical solution of the inverse problem. Results of computational experiments presented for the synthetic noise free and random noisy data in real scale illustrate bounds of applicability of the proposed approach, also its efficiency and accuracy.

1. Introduction

Tsunamis are gravitational i.e. gravity-controlled waves resulting from abrupt large-scale perturbations arising during seaquakes, underwater volcano eruptions, underwater landslides, rock fragment falls, underwater explosions, etc. More than 250 tsunamis were observed in the 20th century, and about 90 percent of all tsunamis are caused by seaquakes. Hence a central component of the early warning system is the quick detection and evaluation of earthquakes. For this aim various ocean measuring equipments positioned on the ocean floor, on buoys or in the form of tide gauges are used to recognize an approaching tsunami. The recent severe tsunamis in Japan (2011), Sumatra (2004), and at the Indian coast (2004) showed that a system producing exact and immediate information about tsunamis is of vital importance. Mathematical modeling and numerical simulations are most used instruments for providing such an information. It is known that some of the

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http://dx.doi.org/10.1016/j.apnum.2014.04.008
0168-9274/

parameters required for direct simulation of tsunamis are bottom relief characteristics and initial perturbation data (tsunami source). The seismic data about the source are usually obtained in a few tens of minutes after the event (the velocity of seismic waves is about five hundred kilometers per minute while the velocity of tsunami waves is less than twelve kilometers per minute). The difference in arrival time of seismic and tsunami waves can be used in operationally refining the tsunami source parameters and modeling the expected tsunami waves on shore. Most suitable physical models related to simulation of tsunamis are based on shallow water equations (see [28] and references therein). For tidal motion, even a very deep ocean may be considered as shallow as its depth will always be much smaller than the tidal wave length [28]. The shallow water equations (also called Saint Venant equations in their unidimensional form) are a set of hyperbolic partial differential equations that describe the flow below a pressure surface in a fluid. For tsunami waveforms analysis numerical as well as analytical methods are intensively used by engineers and mathematicians. In [20] tsunami waveforms have been analyzed analytically, by using Laplace transform in time and Fourier transform in space. An overview of methodologies and techniques related to combining real-time data from tsunameters with numerical model estimates to provide site- and event-specific forecasts for tsunamis in real time, has been presented in [3]. A method for tsunami source estimation by real-time processing of water level records near tsunami sources has been proposed in [29]. A tsunami prediction system with analysis of seismic and surface water data obtained in real time and combined with some tsunami scenarios has been described in [25]. This methodology is based on a set of unit sources to construct the tsunami wave.

In this study we propose a variational approach for identification of an unknown initial tsunami source $q(x, y)$ from measured values $f_m(t) := \eta(x_m, y_m, t)$, $t \in (T_{m_1}, T_{m_2})$, of the free water surface vertical displacement $\eta(x, y, t)$, at a finite number of given points (x_m, y_m), $m = 1, 2, \dots, M$. Note that for inverse problems with a final overdetermination, this approach has been proposed in [9]. The mathematical model used here is based on shallow water equations and leads to the inverse problem of determination of an unknown initial data $q(x, y) := \eta(x, y, 0)$ in the hyperbolic problem from additional information about the solution $\eta(x, y, t)$. Note that the first attempt related to tsunami waveform inversion method has been given in [22]. Here Green's function technique has been used to invert the co-seismic slip from observed tide gauges data, by using a priori information about the tsunami source. As an alternative to S. Satake's technique, an adjoint method for tsunami waveform inversion has been proposed in [19]. This method has the advantage of being able to use the nonlinear shallow water equations also, or other appropriate equation sets, and to optimize an initial state given as a linear or nonlinear function of any set of free parameters. Various discrete adjoint methods have been successfully used in geosciences, in order to forecast tsunami wave propagation (see [2] and references therein).

In order to obtain reasonable results from the computational simulation of a tsunami one needs to determine three important factors: proper governing equations, accurate initial conditions (i.e. the initial sea perturbation) and an optimal computational mesh. Within the linear shallow water equations model, several approaches for selecting optimal mesh sizes have already been proposed. Although initial condition(s) may also be obtained from fault parameters, information from seismic waves is not enough to determine all these parameters in a short and real time. One of the first attempts related to determination of the initial sea perturbation is given in [18]. In this inversion model the data space consists of a given number of waveforms and the model parameter space is represented by the values of the initial water elevation field at a given number of points. The proposed here approach is applicable when the forward problem can be represented in terms of the Green's function. Various inversion techniques based on shallow water equations are proposed also in [10–13,27]. Within the range of a linear shallow water model the problem of reconstruction the time independent tsunami source $\varphi(x, y)$ is considered in [27] as an inverse source problem for the hyperbolic equation $W_{tt} = \operatorname{div}(gh(x, y) \operatorname{grad} W) + f_{tt}(x, y, t)$, under the assumption $f(x, y, t) := H(t)\varphi(x, y)$. As a measured output data, water level records $W(x(s), y(s), t)|_{\gamma(s)}$, along the smooth simple curve $\gamma(s)$ are assumed to be known. Using Picard theorem for compact operators, the solution $\varphi(x, y)$ of the inverse problem is then represented by the singular system $\langle s_j; u_j; v_j \rangle$, and then truncated singular value decomposition (SVD) is applied for numerical inversion of the corresponding input–output operator $\mathcal{A} : \varphi \mapsto \xi$. Note that the collocation method, which permits one to estimate the rank of the input–output operator corresponding to backward parabolic problem, consequently the degree of ill-posedness of the inverse problem has been proposed in [8].

This study presents a mathematical as well as numerical analysis of the inverse problem of identifying an unknown initial tsunami source $q(x, y)$ from limited measured values of the free water surface vertical displacement. The proposed approach is based on the weak solution theory for hyperbolic PDEs and adjoint problem method for minimization of the cost functional

$$J(q) = \sum_{m=1}^{M} \left\| \eta(x_m, y_m, \cdot; q) - f_m \right\|^2_{L^2(T_{m_1}, T_{m_2})}, \tag{1.1}$$

corresponding to the inverse problem. We derive an explicit gradient formula for the cost functional $J(q)$ via the solution of appropriate adjoint problem. This gradient formula is then implemented in the conjugate gradient algorithm (CGA) for the numerical solution of the inverse problem. It is known that the degree of ill-posedness of an inverse problem is determined by the singular value decay rate of the corresponding compact operator, defined by the input–output map. However, an estimation of singular values, in general, is more difficult than the approximate solving of an inverse problem. We adopt here the collocation algorithm, proposed in [8] for backward parabolic problems, to estimate the degree of ill-posedness of the considered inverse problem.