Applied Numerical Mathematics 77 (2014) 31–42

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

Analysis of a multi-frequency electromagnetic imaging functional for thin, crack-like electromagnetic inclusions

Won-Kwang Park

Department of Mathematics, Kookmin University, Seoul, 136-702, Korea

ARTICLE INFO

Article history:
Received 12 November 2012
Received in revised form 1 November 2013
Accepted 1 November 2013
Available online 7 November 2013

Keywords:
Multi-frequency subspace migration imaging algorithm
Thin electromagnetic inclusion
Asymptotic expansion formula
Multi-Static Response (MSR) matrix
Bessel functions
Numerical examples

ABSTRACT

Recently, a non-iterative multi-frequency subspace migration imaging algorithm was developed based on an asymptotic expansion formula for thin, curve-like electromagnetic inclusions and the structure of singular vectors in the Multi-Static Response (MSR) matrix. The present study examines the structure of subspace migration imaging functional and proposes an improved imaging functional weighted by the frequency. We identify the relationship between the imaging functional and Bessel functions of integer order of the first kind. Numerical examples for single and multiple inclusions show that the presented algorithm not only retains the advantages of the traditional imaging functional but also improves the imaging performance.

1. Introduction

Central to this paper is the problem of inverse scattering from thin curve-like electromagnetic inclusion(s) embedded in a homogeneous domain. Generally, the main purpose of inverse scattering problem is to identify characteristics of the target of interest, e.g., shape, location, and material properties from measured scattered field data. Among them, shape reconstruction of extended electromagnetic inhomogeneities with small thickness or perfectly conducting cracks is viewed as a difficult problem because of its ill-posedness and nonlinearity but this attracted the attention of researchers because this problem plays a significant role in many fields such as physics, medical science, and *non-destructive testing of materials*. Consequently, various shape reconstruction algorithms have been reported. However, most of these algorithms are based on Newton-type iteration scheme, which requires addition of a regularization term, complex evaluation of Fréchet derivatives at each iteration step, and *a priori* information of unknown inclusion(s). However, shape reconstruction via the iteration method with a bad initial guess fails if the above conditions are not fulfilled.

On account of this, various non-iterative shape reconstruction algorithms have been developed, such as MUltiple SIgnal Classification (MUSIC) algorithm [5,12,22,24], topological derivative strategy [15,16,18,21], and linear sampling method [7,13]. Recently, multi-frequency based subspace migration imaging algorithm was developed for obtaining a more accurate shape of unknown inclusions. Related articles can be found in [4,10,19,20,23,26] and references therein. However, studies have applied this algorithm heuristically and therefore certain phenomena such as appearance of unexpected ghost replicas cannot be explained. This gave the main impetus for this study to explore the structure of multi-frequency imaging algorithm.

In this paper, we carefully analyze the structure of multi-frequency subspace migration imaging functional by establishing a relationship with the Bessel functions of integer order of the first kind. This is based on the fact that measured boundary

E-mail address: parkwk@kookmin.ac.kr.

0168-9274/$36.00 © 2013 IMACS. Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.apnum.2013.11.001

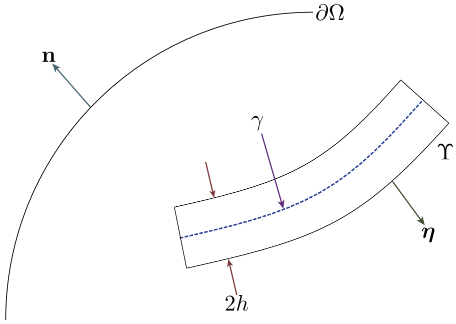

Fig. 1. Two-dimensional thin, curve-like electromagnetic inclusion Υ of thickness $2h$ with supporting curve γ.

data can be represented as an asymptotic expansion formula in the existence of extended, thin inclusion(s). We proceed to explore certain properties of imaging functional and find conditions for good imaging performance. We finally propose an improved imaging functional weighted by power of applied frequencies for producing better results. We carry out a structure analysis of imaging functional, explore a condition of imaging performance, discuss certain properties, and present numerical examples to show its feasibility.

The rest of this paper is organized as follows. In Section 2 we introduce the two-dimensional direct scattering problem and an asymptotic expansion formula for a thin electromagnetic inclusion, and then review the multi-frequency subspace migration imaging functional presented in [3,4,19,20,23]. In Section 3, we discover and specify the structure and properties of the existing multi-frequency subspace migration imaging functional. We then design an improved imaging functional weighted by several applied frequencies and analyze its structure in order to investigate its properties. In Section 4, various numerical examples are exhibited and discussed in order to verify our theoretical results. Finally, in Section 5, conclusion of this paper is presented.

Finally, we would like to mention that although the constructed shape via the proposed algorithm does not match the target shape completely, considering it as an initial guess for an iterative algorithm will be helpful for a successful reconstruction; for details, refer to [1,8,25].

2. Preliminaries

2.1. Direct scattering problem and asymptotic expansion formula

We briefly survey two-dimensional electromagnetic scattering from a thin, curve-like inclusion in a homogeneous domain. For this purpose, let Ω and Υ denote a homogeneous domain with a smooth boundary and a thin inclusion, which is characterized in the neighborhood of a simple, smooth curve γ:

$$\Upsilon = \big\{\mathbf{x} + \rho\boldsymbol{\eta}(\mathbf{x}) \colon \mathbf{x} \in \gamma, -h \leqslant \rho \leqslant h\big\},$$

where h specifies the thickness of Υ. Throughout this paper, we denote $\boldsymbol{\tau}(\mathbf{x})$ *and* $\boldsymbol{\eta}(\mathbf{x})$ as the unit tangential and normal, respectively to γ at $\mathbf{x}$ (see Fig. 1).

Let ε_0 and μ_0 denote the dielectric permittivity and magnetic permeability of Ω, respectively. Similarly, ε and μ *denote* the permittivity and permeability of Υ, respectively. At a given non-zero frequency ω, let $u^{(l)}(\mathbf{x};\omega)$ be the time-harmonic total electromagnetic field that satisfies the following boundary value problem:

$$\begin{cases} \nabla \cdot \left(\dfrac{1}{\mu}\chi(\Upsilon) + \dfrac{1}{\mu_0}\chi(\Omega\backslash\overline{\Upsilon})\right)\nabla u^{(l)}(\mathbf{x};\omega) + \omega^2\big(\varepsilon\chi(\Upsilon) + \varepsilon_0\chi(\Omega\backslash\overline{\Upsilon})\big)u^{(l)}(\mathbf{x};\omega) = 0 & \text{for } \mathbf{x} \in \Omega, \\ \dfrac{1}{\mu_0}\dfrac{\partial u^{(l)}(\mathbf{x};\omega)}{\partial \mathbf{n}(\mathbf{x})} = \dfrac{1}{\mu_0}\dfrac{\partial e^{i\omega\boldsymbol{\theta}_l\cdot\mathbf{x}}}{\partial \mathbf{n}(\mathbf{x})} & \text{for } \mathbf{x} \in \partial\Omega \end{cases} \tag{1}$$

with transmission conditions on $\partial\Omega$ and $\partial\Upsilon$. Here, $\{\boldsymbol{\theta}_l : l = 1, 2, \ldots, Q\}$ is the set of incident directions equally distributed on the unit circle $\mathbb{S}^1$, and $\chi(A)$ *denotes the characteristic function of a set* A. Let $u_0^{(l)}(\mathbf{x};\omega) = e^{i\omega\boldsymbol{\theta}_l\cdot\mathbf{x}}$ be the solution of (1) without Υ. Then, due to the existence of Υ, the following asymptotic expansion formula holds. This formula will contribute to development of the imaging algorithm. A rigorous derivation of this formula can be found in [6].

Theorem 2.1 *(Asymptotic expansion formula). For* $\mathbf{x} \in \gamma$ *and* $\mathbf{y} \in \partial\Omega$*, the following asymptotic expansion formula holds*:

$$u^{(l)}(\mathbf{y};\omega) - u_0^{(l)}(\mathbf{y};\omega) = hu_\gamma^{(l)}(\mathbf{y};\omega) + \mathcal{O}\big(h^2\big),$$

where the perturbation term $u_\gamma^{(l)}(\mathbf{x};\omega)$ *is given by*

$$u_\gamma^{(l)}(\mathbf{y};\omega) = \omega^2 \int\limits_\gamma \left\{\left(\frac{\varepsilon - \varepsilon_0}{\sqrt{\varepsilon_0\mu_0}}\right)u_0^{(l)}(\mathbf{x};\omega)\Lambda(\mathbf{x},\mathbf{y};\omega) + \nabla u_0^{(l)}(\mathbf{x};\omega)\cdot\mathbb{M}(\mathbf{x})\cdot\nabla\Lambda(\mathbf{x},\mathbf{y};\omega)\right\} d\gamma(\mathbf{x}). \tag{2}$$