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A dynamic contact problem in thermoviscoelasticity with two temperatures

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ABSTRACT

This work is concerned with the study of a one-dimensional dynamic contact problem arising in thermoviscoelasticity with two temperatures. The existence and uniqueness of a solution to the continuous problem is established using the Faedo–Galerkin method. A finite element approximation is proposed, a convergence result given and some numerical simulations described.

1. Introduction

In this paper we study a dynamic contact problem between a homogeneous, thermoviscoelastic nonlinear rod and a reactive obstacle based on the theory of thermoelasticity with two temperatures. The difference of this theory with respect to the classical one-temperature theory of thermoelasticity is in the thermal dependence. It takes into account the theory of heat conduction proposed by Chen, Gurtin and Williams [3,4] which relies on two temperatures, namely the conductive and the thermodynamic temperatures, combined with the elasticity theory for deformable bodies. Under this theory, the basic equations (see [4,8,18]) that model the evolution in time of the rod's temperatures and its longitudinal deformation, from a reference configuration given by the interval $I=[0,1]$, and in absence of body forces and heat supply, is

$$u_{tt}(x,t)=\tilde{\sigma}_x(x,t), \quad 0<x<1,\ t>0,$$

$$\eta_t(x,t)=-q_x(x,t), \quad 0<x<1,\ t>0,$$

$$q(x,t)=-\tilde{\phi}_x(x,t), \quad 0<x<1,\ t>0,$$

$$\tilde{\theta}(x,t)=\tilde{\phi}(x,t)-\beta\tilde{\phi}_{xx}(x,t), \quad 0<x<1,\ t>0,$$

$$\eta(x,t)=a u_{xt}(x,t)+\theta(x,t), \quad 0<x<1,\ t>0,$$

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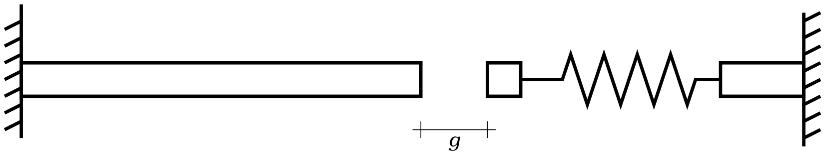

Fig. 1. Contact with an elastic obstacle.

where u is the axial displacement, $\tilde{\sigma}(x,t) = (1 + \int_0^1 u_x(x,t)^2\,dx)u_x(x,t) + \zeta u_{xt}(x,t) - a\tilde{\theta}(x,t)$ the stress, η the entropy, q the heat flux, $\tilde{\theta}$ the thermodynamic temperature and $\tilde{\phi}$ the conductive temperature. Here, viscosity effects are considered and changes in tension resulting from the variations in the length of the rod, as proposed by Woinowsky-Krieger [17], are taken into account.

The above equations are supplemented by the initial conditions

$$\tilde{\phi}(x,0) = \tilde{\phi}_0(x), \qquad u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad 0 < x < 1,$$

and the boundary conditions

$$u(0,t) = 0, \qquad \tilde{\phi}(0,t) = \phi_A, \qquad \tilde{\phi}(1,t) = 0, \quad t > 0,$$
$$\tilde{\sigma}(1,t) = -\frac{1}{\epsilon}\big[u(1,t) - g\big]_+, \quad t > 0.$$

The material parameter $\beta > 0$ is the two temperature parameter and it is called discrepancy. In general, the two temperatures $\tilde{\theta}$, $\tilde{\phi}$ are different [3,16] but when $\beta = 0$ they are equal, and the classical theory of thermoviscoelasticity is recovered. Moreover, $a > 0$ is a constant usually small characteristic of the material and $\zeta > 0$ is the coefficient of viscosity.

The rod is held fixed at the end $x = 0$ while the end $x = 1$ is free to expand and may get in contact with an elastic obstacle, with rigidity $1/\epsilon > 0$, and located at a distance $g > 0$ from the position $x = 1$ (Fig. 1). Both ends are kept with constant conductive temperature.

When ϵ goes to zero, the obstacle becomes rigid and contact is described by the Signorini contact conditions

$$u(1,t) \leqslant g, \qquad \sigma(1,t) \leqslant 0, \qquad \big(u(1,t) - g\big)\sigma(1,t) = 0.$$

The equations of thermodynamics with two temperatures were introduced by Chen, Gurtin and Williams [3,4] for non-simple materials and in recent years the theory of thermoelasticity with two temperatures has received significant attention. The asymptotic behavior of solutions to problems in one and two dimensions was examined by Quintanilla [13]. The propagation of harmonic plane waves under the two-temperatures theory of thermoelasticity was studied by Puri and Jordan [12]. Numerical experiments were performed and the results compared with those obtained under the classical theory of thermoelasticity.

Problems in generalized thermoelasticity with one and two relaxation times and in the context of two temperatures were investigated by Magaña and Quintanilla [10], Youssef [18] and Kumar and Mukhopadhyay [9].

Contact problems are important in industrial applications and were considered by many authors under the classical theory of thermoelasticity. See for example [1,6,7,11,14,15] and references therein. However, to our knowledge, contact problems were not examined in the context of two temperatures. The presence of two temperatures brings new difficulties to the numerical analysis since it seems not to be possible to work only with $\tilde{\phi}$ and u.

It is convenient to transform the problem into one with homogeneous conditions to the conductive temperature. Let $\phi(x,t) = \tilde{\phi}(x,t) + \phi_A(x-1)$ and $\theta(x,t) = \tilde{\theta}(x,t) + \phi_A(x-1)$. Thus, after elimination of η and q, for all $x \in I$ and $t > 0$ we have

$$\phi_t(x,t) - \beta\phi_{xxt}(x,t) - \phi_{xx}(x,t) = -a u_{xt}(x,t), \tag{1}$$
$$u_{tt}(x,t) - \sigma_x(x,t) = 0, \tag{2}$$
$$\theta(x,t) = \phi(x,t) - \beta\phi_{xx}(x,t), \tag{3}$$
$$\phi(x,0) = \tilde{\phi}_0(x) + \phi_A(x-1) \equiv \phi_0(x), \tag{4}$$
$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \tag{5}$$
$$u(0,t) = \phi(0,t) = \phi(1,t) = 0, \tag{6}$$
$$\sigma(1,t) = -\frac{1}{\epsilon}\big[u(1,t) - g\big]_+ \tag{7}$$

where $\sigma(x,t) = (1 + \int_0^1 u_x(x,t)^2\,dx)u_x(x,t) + \zeta u_{xt}(x,t) - a(\phi(x,t) - \beta\phi_{xx}(x,t) - \phi_A(x-1))$.

The next result will be used to deal with the integro-differential term (see [7]).