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A local projection stabilization of fictitious domain method for elliptic boundary value problems

S. Amdouni [a,b], M. Moakher [a], Y. Renard [c,*]

[a] Laboratoire LAMSIN, Ecole Nationale d'Ingénieurs de Tunis, Université Tunis El Manar, B.P. 37, 1002 Tunis-Belvédère, Tunisie
[b] INSA-Lyon, ICJ UMR5208, France
[c] Université de Lyon, CNRS, INSA-Lyon, ICJ UMR5208, LaMCoS UMR5259, F-69621, Villeurbanne, France

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ABSTRACT

In this paper, a new consistent method based on local projections for the stabilization of a Dirichlet condition is presented in the framework of finite element method with a fictitious domain approach. The presentation is made on the Poisson problem but the theoretical and numerical results can be straightforwardly extended to any elliptic boundary value problem. A numerical comparison is performed with the Barbosa–Hughes stabilization technique. The advantage of the new stabilization technique is to affect only the equation on multipliers and thus to be equation independent.

Introduction

The fictitious domain method is a technique allowing the use of regular structured meshes on a simple shaped fictitious domain containing the real domain. Generally, this technique is used for solving elliptic problems in domains with unknown or moving boundary without having to build a body fitted mesh. There exist two main approaches of fictitious domain method. The "thin" interface approach where the approached interface has the same dimension as the original interface. This approach was initiated by V.K. Saul'ev in [29]. In this context, there exist different techniques to take account of the boundary condition: the technique where the fictitious domain mesh is modified locally to take account of the boundary condition (see for instance reference [29,20]); The technique of penalization which allows to conserve the Cartesian mesh of the fictitious domain (see for instance reference [2,17]) and the technique of Lagrange multiplier introduced by R. Glowinski et al. [12,15,17,16] where a second mesh is considered to conserve the Cartesian mesh of the fictitious domain and to take account of the boundary condition.

The second approach of fictitious domain method is the "Spread" interface approach where the approximate interface is larger than the physical interface. The approximate interface has one dimension more than the original one. It was introduced by Rukhovets [28]. For example, the following methods can be found in this group: Immersed boundary method [25,26] and Fat boundary method [21,7].

Recently, fictitious domain methods with a thin interface have been proposed in the context of the extended finite element method (X-FEM) introduced by Moes, Dolbow and Belytscko [23]. Different approaches are proposed in [22,31,6] to directly enforce an inf-sup condition on a multiplier to prescribe a Dirichlet boundary condition. Another possibility is the use of the stabilized Nitsche's method [24] which is close to a penalization technique but preserving the consisting and

* Corresponding author.
E-mail addresses: Saber.Amdouni@insa-lyon.fr (S. Amdouni), maher.moakher@enit.rnu.tn (M. Moakher), Yves.Renard@insa-lyon.fr (Y. Renard).

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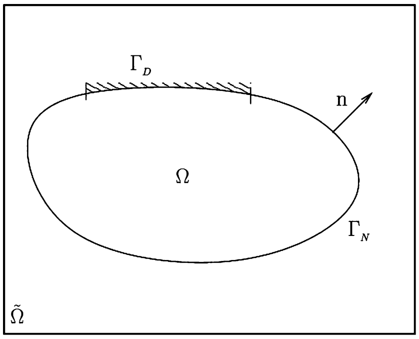

Fig. 1. Fictitious $\tilde{\Omega}$ and real Ω domains.

avoiding large penalty terms that would otherwise deteriorate the conditioning of the system matrix [11]. We can cite also the method introduced in [10] which uses a stabilized Lagrange multipliers method using piecewise constant multipliers and an additional stabilization term employing the inter-element jumps of the multipliers. Finally let us mention [18] where an a priori error estimate for non-stabilized Dirichlet problem is given and an optimal method is developed using a Barbosa–Hughes stabilization (see [3,4]).

In this paper, we perform a study similar to [18] for a local projection stabilization applied to the fictitious domain method inspired by the X-FEM. To our knowledge, this technique was used for the first time by Dohrmann et al. [13]. Recently, this new technique was proposed and analyzed by Burman [9] in the context of the Lagrange finite element method and by Barrenechea et al. [5] in the context of a more classical fictitious domain approach (uncut mesh). The principle of the used local projection stabilization is to penalize the difference of the multiplier with its projection on some pre-defined patches. The advantage of this technique is of at least threefold: the method is consistent, there is no use of mesh other than the (possibly Cartesian) one of the fictitious domain and the additional term concerns only the multiplier and is not model dependent such as the Barbosa–Hughes stabilization technique.

The paper is organized as follows. In Section 1 we introduce the Poisson model problem and in Section 2, the non-stabilized fictitious domain method. We present our new stabilization technique in Section 3 together with the theoretical convergence analysis. Finally, Section 4 is devoted to two and three-dimensional numerical experiments and the comparison with the use of Barbosa–Hughes stabilization technique.

1. The model problem

For the sake of simplicity, the presentation and the theoretical analysis is made for a two-dimensional regular domain Ω, although the method extends naturally to higher dimensions. Let $\widetilde{\Omega} \subset \mathbb{R}^2$ be a fictitious domain containing Ω in its interior (and generally assumed to have a simple shape). We consider that the boundary Γ of Ω is split into two parts Γ_N and Γ_D (see Fig. 1). It is assumed that Γ_D has a nonzero one-dimensional Lebesgue measure. Let us consider the following elliptic problem in Ω:

$$\begin{cases} \text{Find } u : \Omega \mapsto \mathbb{R} \text{ such that:} \\ -\Delta u = f \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \Gamma_D, \\ \partial_n u = g \quad \text{on } \Gamma_N, \end{cases} \tag{1}$$

where $f \in L^2(\Omega)$ and $g \in L^2(\Gamma_N)$ are given data. Considering a Lagrange multiplier to prescribe the Dirichlet boundary condition, a classical weak formulation of this problem reads as follows:

$$\begin{cases} \text{Find } u \in V \text{ and } \lambda \in W \text{ such that} \\ a(u,v) + \langle \lambda, v\rangle_{W,X} = l(v) \quad \forall v \in V, \\ \langle \mu, u\rangle_{W,X} = 0 \quad \forall \mu \in W, \end{cases} \tag{2}$$

where

$$V = H^1(\Omega), \qquad X = \{w \in L^2(\Gamma_D)\colon \exists v \in V,\ w = v_{|\Gamma_D}\}, \qquad W = X',$$

$$a(u,v) = \int_\Omega \nabla u.\nabla v \, d\Omega, \qquad l(v) = \int_\Omega f v \, d\Omega + \int_{\Gamma_N} g v \, d\Gamma,$$

and $\langle \mu, v\rangle_{W,X}$ denotes the duality pairing between W and X, endowed with the following norms:

$$\|v\|_V = \big(a(v,v)\big)^{1/2}, \qquad \|f\|_X = \inf_{v\in V;\, f = v_{|\Gamma_D}} \|v\|_V, \qquad \|\mu\|_W = \sup_{v\in V} \frac{\langle \mu, v\rangle_{W,X}}{\|v\|_V}.$$