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A high order accurate conservative remapping method on staggered meshes

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Abstract

An accurate remapping algorithm is an essential component of the Arbitrary Lagrangian–Eulerian (ALE) methods. Most ALE codes applied to high speed flow problems use a staggered mesh, i.e., all the solution variables except the velocities are cell-centered while the velocities are vertex-centered. In this paper, we present a high order accurate conservative remapping method on staggered meshes by using the idea of essentially non-oscillatory (ENO) schemes. The algorithm is based on the ENO reconstruction and approximate integration. On the staggered mesh, two sets of control volumes are built for the cell-centered conserved quantities including the mass and total energy and vertex-centered quantity—momentum respectively. On each rezoning step, we first reconstruct a polynomial function by the cell averages of mass, energy and momentum on their old control volumes. ENO idea is used to choose the best stencils for reconstruction to avoid oscillation. Then, we integrate the reconstructed functions of the old cells over the rezoned cell. These procedures of remapping ensure the algorithm to have the properties of conservation, high order accuracy and essentially non-oscillatory output. A suite of one and two dimensional examples are given to verify the performance of the algorithm.

Keywords: Remapping; High order accuracy; Conservation; Staggered mesh; ENO reconstruction

1. Introduction

In numerical simulations of fluid flow, traditionally there have been two viewpoints: a Lagrangian framework, in which the mesh moves with the local fluid velocity, and an Eulerian framework, in which the fluid flows through a grid fixed in space. Each framework has its own advantages and disadvantages. In a classic paper [10], Hirt et al., developed the framework in which the grid motion could be determined as an independent degree of freedom, and

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showed that this general framework could be used to combine the best properties of the Lagrangian and Eulerian methods. This class of methods has been termed Arbitrary Lagrangian–Eulerian (ALE). There are two ways to use the ALE framework. One way is to run in a mainly Lagrangian mode, with an occasional rezoning/remapping whenever the grid becomes too distorted. The other way is to rezone and remap for each time step, a strategy termed continuous rezoning. The main components of most ALE algorithms are a Lagrangian phase in which the solution and the grid are updated, a rezoning phase in which the nodes of the computational grid are moved to a more optimal position and a remapping phase in which the Lagrangian solution is transfered to the new grid. We concentrate our attention on the remapping phase in this paper.

The remapping algorithm is an essential and important component of ALE methods. In principle, a good remapping scheme should possess properties such as accuracy, conservation and computational efficiency. In the past thirty years, many authors have worked on optimizing the remapping strategies for cell-centered variables, see for example [5,6, 11,13]. Most of them are however at most second order accurate, especially in two or higher spatial dimensions.

Although there are ALE codes which use a single mesh for all variables, e.g., [2], most ALE codes applied to high speed flow problems use a staggered mesh, where the dynamical variables such as velocities are stored at the nodes whereas the thermodynamic quantities such as density, pressure and energy are stored at the cell centers. This lack of collocation leads to several difficulties, such as the lack of remapping algorithms designed for nodal quantities, and the incompatibility in the remapped values of cell mass and nodal momentum. However, we will not address the second difficulty in this paper. Several first order accurate methods such as YAQUI, SALE and SHALE [4] for remapping nodal quantities were widely used in many old ALE codes. More recently, some second-order remapping methods on staggered meshes have been developed such as the Half-Interval-Shift (HIS) method [3,4], the Recovery and Repair (RR) method [14], and the Subcell Remapping (SR) method [12]. Many of the above mentioned algorithms need the new mesh to be a small displacement of the old mesh, and some of them are constructed by adapting advection algorithms. The distinguished advantage of using an advection algorithm is that it does not require finding the intersections of the Lagrangian mesh (old mesh) with the rezoned mesh (new mesh). However, the connection between the advection equation and conservative remapping does not appear to be well understood; in particular, the underlying assumptions and discretization errors of using advection methods for remapping are not easily identified. Some of the remapping algorithms in the literature, e.g., those in [11,13,8], use the reconstruction plus averaging idea for low order reconstructions. Our work in this paper can be considered as generalizations of these earlier works.

The original idea of ENO reconstruction is proposed by Harten et al. [9], which is the first attempt to obtain a uniformly high-order accurate, yet essentially non-oscillatory reconstruction for piecewise smooth functions. In the following years, ENO schemes have accomplished successful applications in many fields especially with problems containing both shocks and complicated smooth flow structures, see for example [15].

The focus of this paper is on developing a local remapping algorithm on staggered meshes in one and two dimensions. We will attempt to use the ENO reconstruction idea on our remapping strategy so that the algorithm has the framework of arbitrary order of accuracy and has the properties of producing essential non-oscillatory output which are conservative for mass, total energy and momentum. Our remapping algorithm does not require any relationship between the old and the new meshes. It is therefore suitable not only for continuous rezoning but also for occasional rezoning.

We will devise our strategy to:

(1) Define and remap the cell-centered quantities such as density and total energy on their control volumes.
(2) Define and remap the vertex-centered quantity—momentum on its control volumes.
(3) Determine the velocity of nodes on rezoned grid by the remapped values of density and momentum.

An outline of the rest of this paper is as follows. In Section 2, we will describe the individual steps of our remapping procedure and the procedure to obtain the velocity at nodes by the remapped values of density and momentum. In Section 3, we will present one dimensional numerical results, while in Section 4 two dimensional numerical examples are given to verify the performance of our remapping method. In Section 5 we will give concluding remarks.

2. The strategy for density and momentum remapping

In order to save space we will only describe the procedure in two dimension. The procedure in one dimension is similar but much simpler. We consider a two-dimensional computational domain Ω, which is assumed to be a