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Applied Numerical Mathematics 57 (2007) 811–820

APPLIED NUMERICAL MATHEMATICS

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A fast iterative solver for scattering by elastic objects in layered media

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Available online 23 August 2006

Abstract

We developed a fast iterative solver for computing time-harmonic acoustic waves scattered by an elastic object in layered media. The discretization of the problem was performed using a finite element method with linear elements based on a locally body-fitted uniform triangulation. We used a domain decomposition preconditioner in the iterative solution of the resulting system of linear equations. The preconditioner was based on a cyclic reduction type fast direct solver. The solution procedure reduces GMRES iterates onto a sparse subspace which decreases the storage and computational requirements essentially. The numerical results demonstrate the effectiveness of the proposed approach for two-dimensional domains that are hundreds of wavelengths wide and require the solution of linear systems with several millions of unknowns.

1. Introduction

We consider a numerical method for computing time-harmonic acoustic waves scattered by an elastic object Ω in layered fluid. The proposed method is efficient when the interfaces between layers are nearly horizontal, for example, rippled horizontal interfaces. One application for such problems is the detection of hazardous or/and lost objects buried in sediment. For this purpose it is useful to have a numerical approximation which sufficiently accurately predicts backscatter by such targets.

Our model problem in a rectangular domain Π is shown in Fig. 1. The density of the medium ρ and the speed of sound c are assumed to be constant in both fluid layers. The described approach can be generalized for the case that ρ and c are depth-dependent functions in both fluid layers, but we will not consider this in here. We have for the pressure p in the fluid and for the displacement u in the elastic object Ω the partial differential equation model

$$\begin{aligned}
&\nabla\cdot\frac{1}{\rho}\nabla p+\frac{k^2}{\rho}p=g \quad \text{in } \Pi\setminus\Omega,\\
&\frac{1}{\rho}\frac{\partial p}{\partial n}=\omega^2 u\cdot n, \qquad -pn=\sigma(u)n \quad \text{on } \partial\Omega,\\
&\nabla\cdot\sigma(u)+\omega^2\rho u=0 \quad \text{in } \Omega,\\
&\mathcal{B}p=0 \quad \text{on } \partial\Pi,
\end{aligned} \tag{1.1}$$

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doi:10.1016/j.apnum.2006.07.020

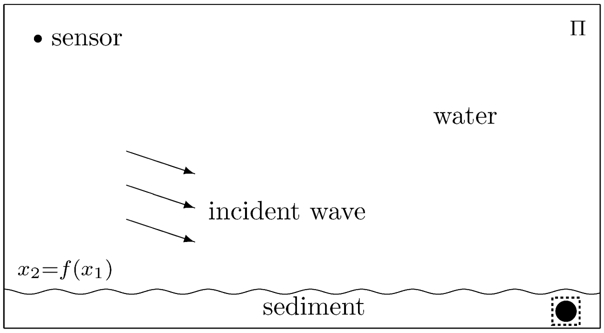

Fig. 1. A model problem with an elastic object Ω in sediment; the dashed line near Ω decomposes the domain Π into two subdomains which will be employed in the solution procedure.

where ω is the angular frequency, $k=\omega/c$ is the wave number, g is an acoustic source term, n denotes the unit outward normal vector of $\partial\Omega$, and $\sigma(u)$ is the stress tensor. The operator $\mathcal{B}$ corresponds to a second-order absorbing boundary condition which is a generalization of the ones in [1,7] for nonhomogenous media. For more discussion on scattering problems in layered media see [6], for example.

With higher frequencies a finite element discretization leads to very large systems of linear equations. Often two-dimensional problems have millions of unknowns. It might be possible to solve these problems using a LU factorization with a nested dissection reordering of unknowns, but this approach cannot be used for three-dimensional problems which can have billions of unknowns. For this reason, we consider the iterative solution of these problems. In the right preconditioned GMRES method we employ a domain decomposition preconditioner based on an algebraic fictitious domain approach [12,14–16,19,20,22,26]. The preconditioning of discretized scattering problems in layered media without an object has been considered in [8,9,24,25,28], for example, and with an object in [17]. The domain decomposition method introduced in [13] and employed in [3] for computing electromagnetic scattering by coated objects is based on a similar approach to the one considered in here. In [13] the electromagnetic scatterer is perfectly conducting with a dielectric coating layer which would corresponds to a sound-soft acoustic scatterer with a coating layer having different density and speed of sound. The approach in here uses a Schur complement preconditioner for far field while the method in [13] used an algebraic fictitious domain approach for this. In this paper, we consider more complicated scattering problems and use a more straightforward block preconditioner than in [13].

We discretize (1.1) using linear finite elements on uniform rectangular meshes which are locally adapted to the wavy sediment interface and the surface of the object. An algorithm to generate such meshes is described in [5], for example. In our solution procedure we employ a domain decomposition in which the near field subdomain is the interior of the dashed box in Fig. 1 and the far field subdomain is the rest of the rectangle Π. For the second subdomain we construct a preconditioner based on a separable matrix obtained by discretizing perfectly vertically layered media without an object. Linear systems with such matrices can be solved efficiently using fast direct methods [29,30,34]. Since the media is vertically layered with a wavy interface, our preconditioner coincides with the system matrix except for the rows corresponding to unknowns near-by the interfaces. Due to this we can reduce iterations onto a small sparse subspace as has been shown in [19,20]. This reduction makes our preconditioner much more efficient as our numerical examples demonstrate.

2. Finite element discretization

For two-dimensional problems, we use a generalization of the second-order absorbing boundary condition in [1] on the truncation boundary $\partial\Pi$, given by

$$\begin{aligned}
&\frac{1}{\rho}\frac{\partial p}{\partial n}=i\frac{k}{\rho}p+i\frac{1}{2k}\frac{\partial}{\partial s}\frac{1}{\rho}\frac{\partial p}{\partial s} \quad \text{on } \partial\Pi,\\
&\frac{1}{\rho}\frac{\partial p}{\partial n}=i\frac{3k}{4\rho}p \quad \text{at } C,
\end{aligned} \tag{2.1}$$