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Global convergence of a tri-dimensional filter SQP algorithm based on the line search method ☆

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Abstract

In this paper, we propose a new filter line search SQP method in which the violations of equality and inequality constraints are considered separately. Thus the filter in our algorithm is composed by three components: objective function value, equality and inequality constraints violations. The filter with three components accepts reasonable steps flexibly, comparing that with two components. The new filter shares some features with the Chin and Fletcher's approach, namely the "slanting envelope" and the "inclusion property". Under mild conditions, the filter line search SQP method is proven to be globally convergent. Numerical experiments also show the efficiency of our method.

MSC: 65K05; 90C30

Keywords: Line search method; Backtracking; Constrained optimization problem; SQP; Filter

1. Introduction

Fletcher and Leyffer [9] proposed the filter method to solve nonlinear programming (NLP) problems as an alternative to the traditional merit function approach. The underlying concept of filter is fairly simple, being based on the multi-objective optimization, that is the trial point is accepted provided there is a sufficient decrease of the objective function or the constraint violation function. In addition, the computational results presented in Fletcher and Leyffer [9] are also very encouraging. This topic got high importance in recent years (see [5,11,14,21,22]). Trust region filter sequential quadratic programming (SQP) methods have been studied by Fletcher, Leyffer and Toint in [10] and by Fletcher, Gould, Leyffer, Toint and Wächter in [8]. In this latter paper, an approximate solution of the QP subproblem is computed and the trial step is decomposed into normal and tangential components. Moreover, Gould and Toint proposed in [18] a hybrid trust region filter SQP algorithm. In all these papers only the global convergence of the proposed methods is analyzed. On the other hand, in [26], Ulbrich studied the local convergence of a trust region filter

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SQP method. Anyway, the components in the filter adopted in [26], differs from those in [9,10,8]. It should be underlined that the filters approach has been used also in conjunction with the line search strategy (see Wächter and Biegler [29,30]); with interior point methods (see Benson, Shanno and Vanderbei [4]; Ulbrich, Ulbrich and Vicente [27]) and with the pattern search method (see Audet and Dennis [1]). Finally, the filter's idea has been employed to solve least squares problems and nonlinear equations [15] and unconstrained optimization problems [16]. In these latter papers, as well as in the code FILTRANE [17], multidimensional filters are employed.

The filter methods, compared to the traditional penalty function methods in which the adjustment of the penalty parameter can be problematic, may make the trial steps accepted more easily. The numerical results [9,17] are more efficient. However, most of them use the bi-dimensional filter methods, and only a few of them use the multidimensional filter [16,15,17] solving the unconstrained optimization problems.

In this paper, inspired by [15], we propose a tri-dimensional filter method based on the line search technique. The underlying idea of filter is to interpret the NLP problem as a bi-objective optimization problem with two conflicting purposes: minimizing the constraint violation and the objective function. The formal filter in [9] consists of two parts: the objective function's value and the constraint violation. It considers all the constraints together and defines only one constraint violation. However, each constraint may have its own behavior. For example, some constraints may be highly nonlinear, while some others are nearly linear. We split the constraints into equality and inequality constraints. However, the partition used does not influence the global convergence analysis. We define the corresponding constraint violations and drive the equality and inequality constraint violations to zero independently. Thus, the new filter consists of three values: objective function value, equality and inequality constraint violations. This strategy admits more flexibility in accepting steps as compared with the bi-dimensional filter methods.

Furthermore, the filter we proposed here contains a "slanting envelope" originally proposed in [5,9]. The new tri-dimensional filter takes advantage of the "inclusion property" which will be described in Subsection 2.1 in detail. In fact, if the filter is updated, then the space of unacceptable points for the new filter includes that for the old filter. We remark that this is not guaranteed for the filters in [9,29].

This paper is organized as follows. In Section 2 the algorithm is developed and its key ingredients are described. Namely, the decrease conditions for the objective function, the equality and inequality constraint violations. In Section 3 we prove that under suitable conditions, the algorithm is well defined and globally convergent. In Section 4 we illustrate the implementation issues and some of the obtained numerical results.

2. Algorithm

In this paper, we consider an NLP problem of the following form:

$$\text{(P)} \quad \begin{cases} \min\limits_{x\in R^n} & f(x) \\ \text{s.t.} & c_{\mathcal{E}}(x) = 0, \\ & c_{\mathcal{I}}(x) \leqslant 0, \end{cases} \tag{2.1}$$

where $f : R^n \to R$, $c_{\mathcal{E}}(x) = (c_1(x), c_2(x), \dots, c_p(x))^T$, $c_{\mathcal{I}}(x) = (c_{p+1}(x), c_{p+2}(x), \dots, c_q(x))^T$ are twice continuously differentiable functions, $\mathcal{E} = \{1, \dots, p\}$, $\mathcal{I} = \{p+1, \dots, q\}$.

The KKT conditions for problem (P) are

$$\begin{cases} \nabla f(x) + \nabla c_{\mathcal{E}}(x)\lambda + \nabla c_{\mathcal{I}}(x)\mu = 0, \\ c_{\mathcal{E}}(x) = 0, \quad c_{\mathcal{I}}(x) \leqslant 0, \\ c_{\mathcal{I}}(x)^T \mu = 0, \\ \mu \geqslant 0, \end{cases} \tag{2.2}$$

where $\lambda \in \mathbb{R}^p$ and $\mu \in \mathbb{R}^q$ are the Lagrange multipliers corresponding to equality and inequality constraints, and $\nabla c_{\mathcal{E}}$ and $\nabla c_{\mathcal{I}}$ are the Jacobian of $c_{\mathcal{E}}$ and $c_{\mathcal{I}}$, respectively.

Let x_k be the current solution at iteration k. The search direction d_k is obtained by solving the following quadratic programming (QP) problem:

$$\text{QP}(x_k) \quad \begin{cases} \min\limits_{d\in R^n} & \nabla f(x_k)^T d + \frac{1}{2} d^T B_k d \\ \text{s.t.} & c_{\mathcal{E}}(x_k) + \nabla c_{\mathcal{E}}(x_k)^T d = 0, \\ & c_{\mathcal{I}}(x_k) + \nabla c_{\mathcal{I}}(x_k)^T d \leqslant 0, \end{cases}$$