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Extension of generalized recursive Tau method to non-linear ordinary differential equations

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Abstract

In a recent paper, we reported a generalized approximation technique for the recursive formulation of the Tau method. This paper is concerned with an extension of that discourse to non-linear ordinary differential equations. The numerical results show that the method is effective and accurate.

Keywords: Lanczos Tau method; Chebyshev polynomials; Initial value problems; Canonical polynomial; Ordinary differential equations

1. Introduction

In 1981, Ortiz and Samara [1] proposed an operational technique for the numerical solution of non-linear ordinary differential equations with some supplementary conditions based on the Tau method [2]. Recently, considerable work has been done both in the development of the technique, its theoretical analysis and numerical applications. The technique has been described in a series of papers [3–6,1,12–15], for the case of linear ordinary differential eigenvalue problems. Yisa and Adeniyi [7] reported the construction of generalized canonical polynomials while Issa and Adeniyi [8,16] reported generalized approximation for the recursive formulation of the Tau method for the solution of ordinary differential equations, their earlier works are further extended to non-linear ordinary differential equations.

2. Recursive formulation of Tau approximant

In this section, we review the Tau approximant for the recursive form (see [8]) using the generalized Canonical polynomials $Q_n(x)$ (see [7]) to solve the mth order ordinary differential equation of the form:

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$$Ly(x) \equiv \sum_{r=0}^{m} \left(\sum_{k=0}^{N_r} P_{r,k} x^k \right) y^{(r)}(x) = \sum_{r=0}^{\sigma} f_r x^r, \quad a \le x \le b \tag{2.1a}$$

$$L^* y(x_{rk}) \equiv \sum_{r=0}^{m-1} a_{rk} y^{(r)}(x_{rk}) = \alpha_k, \quad k = 1(1)m \tag{2.1b}$$

by seeking an approximant

$$y_n(x) = \sum_{r=0}^{n} a_r x^r, \quad r < +\infty$$

of $y(x)$ which is the exact solution of the corresponding perturbed system

$$Ly_n(x) = \sum_{r=0}^{\sigma} f_r x^r + H_n(x) \tag{2.2a}$$

$$L^* y_n(x_{rk}) = \alpha_k, \quad k = 1(1)m \tag{2.2b}$$

where α_k, f_k, $P_{r,k}$, N_r; $r = 0(1)m$, $k = 0(1)N_r$ are real integers, $y^{(r)}$ denote the derivatives of order r of $y(x)$, the perturbation term $H_n(x)$ in (2.2a) is defined by:

$$H_n(x) = \sum_{i=0}^{m+s-1} \tau_{i+1} T_{n-m+i+1}(x) = \sum_{i=0}^{m+s-1} \tau_{i+1} \sum_{r=0}^{n-m+i+1} C_r^{(n-m+i+1)} x^r \tag{2.2c}$$

and $C_r^{(n)}$ is the coefficient of x^r in the nth degree Chebyshev polynomial $T_n(x)$; that is,

$$T_n(x) = \cos\left(n \cos^{-1} \left\{ \frac{2x - a - b}{b - a} \right\} \right) \equiv \sum_{r=0}^{n} C_r^{(n)} x^r.$$

The τ's are fixed parameters to be determined and s, the number of overdetermination of (2.1a), is defined by:

$$s = max \{ N_r - r > 0 \,|\, 0 \le r \le m \}.$$

For different orders m and s (that is $m = 1, 2, \ldots$ and $s = 1, 2, \ldots$) we have

$$y_n(x) = \sum_{r=s}^{\sigma} f_r q_r(x) + \sum_{i=0}^{m+s-1} \tau_{i+1} \sum_{r=s}^{n-m+i+1} C_r^{(n-m+i+1)} q_r(x). \tag{2.3a}$$

Assume $Q_r(x) = P_r = 1, r = 0(1)(s-1)$ and

$$\sum_{i=0}^{m+s-1} \tau_{i+1} \sum_{r=0}^{n-m+i+1} C_r^{(n-m+i+1)} P_r + \sum_{r=0}^{\sigma} f_r P_r = 0 \tag{2.3b}$$

where Eq. (2.3b) is the coefficient of undetermined Canonical polynomials, $q_n(x) = Q_n(x) - P_n$,

$$P_n = \frac{-1}{\sum_{k=0}^{m} k! \binom{n-s}{k} P_{k,k+s}} \left\{ \sum_{k=1}^{m} \left(\sum_{j=k}^{m} j! \binom{n-s}{j} P_{j,j-k} \right) P_{n-s-k} + \sum_{k=0}^{s-1} \left(\sum_{j=0}^{m} j! \binom{n-s}{j} P_{j,j+k} \right) P_{n-s+k} \right\} \tag{2.3c}$$