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Elastic interaction of a screw dislocation with a double-coated cylinder

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Abstract

We study the problem of a screw dislocation interacting with a double-coated cylindrical inclusion in a dissimilar matrix. Using an elastostatic image method, we determine the displacement field in each material phase and obtain the force of interaction between the inhomogeneity and the dislocation in the form of a rapidly convergent infinite series. Numerical examples are presented to illustrate the effect of double coating on the force of interaction. It is found that, for certain material combinations, the presence of a double-coated cylinder gives rise to two equilibrium positions of the dislocation. This result is significant as it indicates that multiple equilibrium positions are possible for multiply-coated inclusions interacting with a screw dislocation.

Keywords: Screw dislocation; Double coating; Cylindrical inclusion; Elastostatic image; Interaction force

1. Introduction

The coated inclusion problem plays an important role in the development of advanced composite materials for high technology applications. Often, the composite material consists of a continuous matrix reinforced by inclusions of a different phase. In the presence of material defects, such as dislocations, stress concentrations may build up around an inclusion with adverse consequences. One way of reducing the damage caused by stress concentrations around an inclusion is to protect its surface with a layer of coating of a different material. For instance, a coating layer may be introduced at the design stage to reduce the potential for fibre fracture initiated by matrix cracking [1] or to act as a thermal barrier in high temperature systems [2]. A rigorous analysis of the interaction of dislocations with coated inclusions is necessary to provide valuable insight into the mechanical behaviour of high technology composite materials.

A fair amount of work has been devoted to the case of a screw dislocation interacting with a single-coated cylindrical inclusion (see, for example, Xiao and Chen [3,4], Liu et al. [5], Jiang et al. [6], Sudak [7], Liu et al. [8,9], Feng et al. [10]). However, in some applications, more than one layer of coating may be required to achieve the desired qualities of a product. An example is the use of dual-layer protective coatings to enhance the strength of optical fibers for

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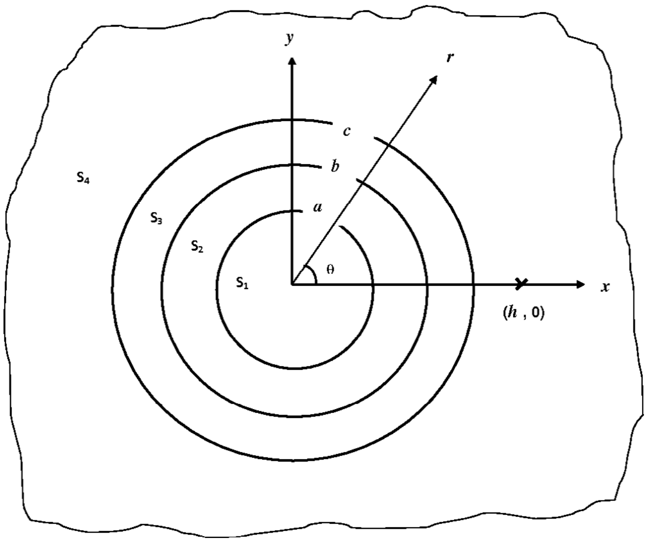

Fig. 1. A double-coated cylindrical inclusion in a matrix containing a screw dislocation.

telecommunications [11]. Therefore, the study of the elastic interaction of multi-coated inclusions with dislocations is a subject of current scientific interest. Honein et al. [12] has determined the force acting on a screw dislocation in the presence of a multi-layered circular inclusion in terms of a rapidly convergent Laurent series whose coefficients are related to those of the complex potential of a corresponding homogeneous problem. More recently, Kuo [2] examined the effect of coating on the magnetoelectroelastic potential in a multicoated elliptic fibrous composite with piezoelectric and piezomagnetic phases, Wang and Schiavone [13] considered the problem of a double-coated inclusion of arbitrary shape when the matrix is subjected to remote uniform anti-plane stresses, Wang and Zhou [14] investigated the long-range interaction of an edge dislocation with multiple inclusions of arbitrary shape while Bonfoh et al. [15] studied the effective properties of composite materials with multi-coated inclusions. Wang and Zhou [16] investigated the conditions for the existence of equilibrium positions for a screw dislocation interacting with a multi-coated inclusion and used the results to attempt the design of near-cloaking mulitcoated structures. The complex variable formulation of anti-plane elastostatics was used for these studies.

In this paper, we study the elastic interaction of a screw dislocation with a double-coated cylindrical inclusion which is embedded in a dissimilar matrix. The objective of this study is to determine the displacement field in each material phase and use the result to examine the effect of double coating on the interaction force between the inhomogeneity and the dislocation. As noted in a recent review of works on inclusions [17], most solutions to inclusion-dislocation interaction problems employ the complex variable method to determine the elastic fields. Here, we shall use the principle of images, in conjunction with Fourier series representations, to construct an analytical solution for the displacement field. We shall, also, discuss the mobility and stability of the dislocation for various material combinations. It will be shown that, unlike the complex variable method, the proposed method considerably reduces the amount of mathematical rigour involved in determining the necessary perturbation fields.

2. Statement of the problem

The problem we analyze is that of a screw dislocation in a solid containing a double-coated cylindrical inclusion. In cylindrical polar coordinates (r, θ, z), the solid under consideration has four phases of shear moduli μ_i $(i = 1, 2, 3, 4)$ which occupy, respectively, four regions S_i defined for $0 \le \theta \le 2\pi$ and $-\infty < z < \infty$ by

$$S_1 : 0 \le r < a, \qquad S_2 : a \le r < b, \qquad S_3 : b \le r < c, \qquad S_4 : c \le r < \infty. \tag{1}$$

The four phases are assumed to be perfectly bonded at the three interfaces $r = a, b, c$. A screw dislocation of Burger's vector $(0, 0, b_z)$ is located at a point $(h, 0)$ in the matrix S_4. A schematic diagram of the problem under consideration is presented in Fig. 1.