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A family of Continuous Third Derivative Block Methods for solving stiff systems of first order ordinary differential equations

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Abstract

This paper presents a family of Continuous Third Derivative Block Methods (CTDBM) of order $k + 3$ for the solution of stiff systems of ordinary differential equations. The approach uses the collocation and interpolation technique to generate the main Continuous Third Derivative method (CTDM) which is then used to obtain the additional methods that are combined as a single block methods. Analysis of the methods show that the method is L-stable up to order eight. Numerical examples are given to illustrate the accuracy and efficiency of the proposed method.

Keywords: Third derivative; Block method; Stability; ODE

1. Introduction

Consider the first order ordinary differential equation

$$y' = f(t, y)\ y(t_0) = y_0. \tag{1}$$

Eq. (1) occurs in several areas of engineering, science and social sciences. It is well known that some of these problems have proved to be either difficult to solve or cannot be solved analytically, hence the necessity of numerical techniques for such problems remains vital. Many physical problems are modeled into first order problem (1), while those modeled in higher order differential equations are either solved directly or solved by first reducing them to system of first-order differential equations. There are various methods available for solving systems of first order IVPs [1,2]. Linear multistep methods for the solution of (1) have been developed varying from discrete linear multistep method to continuous ones. Continuous linear multistep methods have greater advantages over the discrete methods such that they give better error estimation, provide a simplified form of coefficients for further evaluation at different points, and provides solution at all interior points within the interval of integration, (see [3,4]). These methods are first derivative methods that are implemented in predictor corrector mode, and Taylor series expansion are adopted

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to supply starting values. The setback of the predictor–corrector methods are that they are very costly to implement, longer computer time and greater human effort and reduced order of accuracy, which affect the accuracy of the method. Second derivative methods have been proposed by Enright [5], Ismail [6], Hojjati [7]. Recently Ezzeddine and Hojjati [8] proposed third derivative method of order $k+3$. These methods are implemented in a step-by-step fashion in which on the partition Γ, an approximation is obtained at t_{n+1} only after an approximation at t_n has been computed, where $\Gamma : a = t_0 < t_1 < \cdots < t_N = b$, $t_{n+1} = t_n + hn = 0, 1, \ldots, N-1$ $h = \frac{b-a}{N}$ is the constant step-size of the partition of Γ, N is a positive integer, and n is the grid index. High-order continuous third derivative formulas with block extensions have also been proposed by Jator et al. [9] for the direct solution of the general second order ordinary differential equations. In this paper, a family of Continuous Third Derivative Block Method (CTDBM) that will not only be self starting but are also of good accuracy and have stability properties for effective and efficient solution of stiff system of ordinary differential equations of the form (1) is proposed.

2. Derivation of the method

In this section, a k-step third derivative method of the form

$$y_{n+k} = y_{n+k-1} + h\sum_{j=0}^{k} \alpha_j(t) f_{n+j} + h^2 \beta_k g_{n+k} + h^3 \eta_k \gamma_{n+k} \tag{2}$$

is developed for (1) on the interval from t_n to t_{n+k}.

The initial assumption is that the solution on the interval $[t_n, t_{n+k}]$ is locally approximated by the polynomial

$$Y(t) = \sum_{j=0}^{k+3} \tau_j t^j, \tag{3}$$

where τ_j are unknown coefficients. Since this polynomial must pass through the interpolation points (t_{n+k-1}, y_{n+k-1}) and the collocation points $(t_n, y_n, t_{n+1}, y_{n+1}), \ldots (t_{n+k}, y_{n+k})$, we require that the following $(k+4)$ equations must be satisfied.

$$\sum_{j=0}^{k+3} \tau_j t^j = y_{n+i}, \quad i = k-1. \tag{4}$$

$$\sum_{j=0}^{k+3} j\tau_j t^{j-1} = f_{n+i}, \quad i = 0, \ldots, k. \tag{5}$$

$$\sum_{j=0}^{k+3} j(j-1)\tau_j t^{j-2} = g_{n+i}, \quad i = k. \tag{6}$$

$$\sum_{j=0}^{k+3} j(j-1)(j-2)\tau_j t^{j-3} = \gamma_{n+k}, \quad i = k. \tag{7}$$

The $(k+4)$ undetermined coefficients τ_j are obtained by solving Eqs. (3)–(6) and are then substituted into (2). After some algebraic simplification the continuous representation of the third derivative method obtained is given in the form

$$Y(t) = y_{n+k-1} + h\sum_{j=0}^{k} \alpha_j(t) f_{n+j} + h^2 \beta_k(t) g_{n+k} + h^3 \eta_k(t) \gamma_{n+k} \tag{8}$$

where $\alpha_j(t)$, $j = 0, 1, \ldots, k$, $\beta_k(t)$, and $\eta_k(t)$, are continuous coefficients k is the step number, and h is the chosen step-length. We assume that $y_{n+j} = Y(t_n + jh)$ is the numerical approximation to the analytical solution $y(t_{n+j})$, $y'_{n+j} = f(t_{n+j}, y_{n+j})$ is an approximation to $y'(t_{n+j})$, $g_{n+k} = \frac{df}{dt}(t_{n+k}, y(t_{n+k}))$, and $\gamma_{n+k} = \frac{d^2 f}{dt^2}$ $(t_{n+k}, y(t_{n+k}))$.