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Numerical solution of heat conduction problems using orthogonal collocation on finite elements

Shelly Arora[a,b,*], Inderpreet Kaur[a]

[a] Department of Mathematics, Punjabi University, Patiala 147002, Punjab, India
[b] Department of Mathematics and Statistics, College of Arts and Science, University of South Florida, E. Fowler Ave, Tampa 33620, FL, USA

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Abstract

Technique of orthogonal collocation along with finite elements has been presented to solve the linear and non linear heat conduction problems numerically. Choice of Lagrangian interpolation polynomials as base function has been opted to discretize the trial function. Error analysis has been discussed in terms of element size for both the linear and non linear problems. Proposed technique has been applied on different types of linear and non linear heat conduction problems and the numerical values are plotted using 2D and 3D graphs.

Keywords: Orthogonal collocation; Finite elements; Lagrangian interpolation

1. Introduction

Study of advection diffusion equations is an important part of the mass transport problems in porous media. These problems have numerous applications in the field of Physics, Chemical engineering, control system, Computer science, mass transfer, fluid flow problems etc. These problems have aroused the interest of mathematicians as well as engineers while studying the porous media problems. Various investigators have studied the diffusion dispersion problems in different forms [1–7].

Different investigators have proposed different techniques like finite difference method [8–11], homotopy perturbation method [12,13], spline collocation [14–17], Adomain decomposition method [18] etc. In present study, the technique of orthogonal collocation using Lagrangian basis along with finite elements has been proposed to study the behaviour of heat conduction problems numerically.

Consider a heat conduction problem in which, the volumetric concentration $y(x, \tau)$ at $x \in \Omega$ in one dimensional moving fluid with speed $\beta(x)$ and diffusion coefficient ε in longitudinal direction at time $\tau \geq 0$ is of the type:

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* Corresponding author at: Department of Mathematics, Punjabi University, Patiala 147002, Punjab, India.
E-mail address: aroshelly@gmail.com (S. Arora).

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$$\frac{\partial y}{\partial \tau} = \varepsilon \frac{\partial^2 y}{\partial x^2} - \beta(x)\frac{\partial y}{\partial x} - f(y); \qquad (x, \tau) \in \Omega \times (0, 1) \tag{1}$$

$$\text{with} \quad k_1 y - \varepsilon \frac{\partial y}{\partial x} = 0 \qquad \text{at } x = 0 \tag{2}$$

$$k_2 y - k_3 \frac{\partial y}{\partial x} = 0 \qquad \text{at } x = 1 \tag{3}$$

$$\text{Initially } y(x, 0) = g(x) \tag{4}$$

where $f(y)$ defines the implicit function of y and k_1, k_2 and k_3 are arbitrary positive constants with $\beta(x) > 0,\ \forall x \in \Omega$.

Aim of the present study is to analyse the behaviour of problem defined in Eqs. (1)–(4) numerically, with given boundary conditions using the technique of orthogonal collocation on finite elements (OCFE). This technique is the combination of weighted residual and variational principle.

The structure of the paper deals with the detailed description of OCFE in Section 2, error analysis is discussed in Section 3. Application of the technique is shown in Section 4 by applying the technique on different types of linear and nonlinear heat conduction problems and finally the conclusion part is discussed in Section 5.

2. Orthogonal collocation on finite elements

Orthogonal collocation is one of the weighted residual methods which is used to descritize the initial and boundary value problems. In collocation technique an unknown function $y(x, \tau)$ is approximated to satisfy the differential equation $\Im^V(y) = 0$ along with the boundary conditions $\Im^B(y) = 0$, where B is the boundary adjoining the volume V. The solution function y is approximated using a trial function $\overline{y}$ which is a linear combination of series of orthogonal polynomials. The residual function $\Re(x, \tau)$ is defined over the volume V along with the boundary B.

In the principle of collocation, to minimise the error, inner product of residual function to the weight function of given base polynomial is set equal to zero at collocation points, *i.e.*, $\langle \Re(x, \tau),\ W(x)\rangle = 0$. It forces the residual to be equal to zero at the collocation points.

In present study the principle of orthogonal collocation is studied in conjunction with finite elements, *i.e.*, orthogonal collocation on finite elements (OCFE). The domain of interest is called global domain. To apply collocation, this global domain is divided into small subdomains called elements. After applying collocation, the resulting system of equations is compiled to obtain the desired solution.

The first step in OCFE is the approximation of the trial function. Different investigators have followed different type of polynomials to approximate the trial function such as cubic Hermite [19–21], quintic Hermite [22], cubic B-splines [15,17], Lagrangian [23–27] etc. In present study, the Lagrangian interpolating polynomials have been chosen as base functions to discretize the approximating function. Lagrangian interpolating polynomials have the kronecker property due to which the approximating function simplifies at collocation points. Due to this kronecker property the trial function and its tangent are assumed to be continuous at node points by introducing the continuity condition defined below:

$$y\,(x_{i-}) = y(x_{i+}) \tag{5}$$

$$\left.\frac{dy}{dx}\right|_{x_{i-}} = \left.\frac{dy}{dx}\right|_{x_{i+}}. \tag{6}$$

2.1. Collocation points

Next step in collocation technique is the choice of collocation points. It is the most important part of collocation technique as wrong choice of collocation points may lead to divergent results. Preferably the zeros of base orthogonal polynomial are used as collocation points to keep the error minimum. Different investigators have followed the zeros of different types of polynomials such as Legendre [28,29] and Chebyshev [23,27] as collocation points. The discretization end points are fixed as 0 and 1. In present study, the zeros of shifted Jacobi polynomials $P_n^{(a,b)}(x)$ have been taken as collocation points for $a = b = 0$.