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The flow of second grade fluid over a stretching sheet with variable thermal conductivity and viscosity in the presence of heat source/sink

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Abstract

Steady two-dimensional non-Newtonian second grade fluid is studied under the influence of temperature dependent viscosity and thermal conductivity. The viscosity is assumed to vary inversely as linear function of temperature while the thermal conductivity varies directly as linear function of temperature. Also, effects of radiative heat, viscous dissipation and heat source/sink are considered in the energy equation. The basic governing partial differential equations for the velocity and temperature are transformed to ordinary differential equations (ODEs) using appropriate similarity variables. These coupled nonlinear ODEs have been solved approximately subject to appropriate boundary conditions by Runge–Kutta shooting technique. The quantitative effects of emerging dimensionless physical parameters on the velocity, temperature, skin friction and heat transfer rate are displayed graphically. The numerical investigation of the variable thermo-physical properties of a second grade fluid over a stretching sheet provides an extension to previous work.

Keywords: Second grade fluid; Thermal conductivity; Stretching sheet; Variable viscosity; Heat radiation

1. Introduction

The study of boundary layer behaviour of fluids with stretching of elastic sheet from a fixed point with or without varying velocity has applications in manufacturing processes; for instance paper making, polymer processing, food processing, oil recovery etc. The popular Navier–Stokes fluid has been used to model and analyze various aspect of stretching sheet by many researchers (see Refs. [1–6] for details). It is however known that Newtonian fluid has limited industrial applications. Hence, the need for non-Newtonian fluids with particular emphasizes on the second grade fluid.

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Second grade fluid flows with constant viscosity over a stretching surface are the subject of considerable attention in many boundary layer flows. Mamaloukas et al. [7] demonstrated the likeness of the free-parameter method and the separation of variables method to determine the exact solution for the problem of boundary layer flow of an incompressible viscoelastic second grade fluid along a stretching sheet. Hsiao [8] investigated MHD flow of a second grade fluid near stretching sheet with electric and magnetic dissipation and non-uniform heat source/sink. His investigation revealed that the electric dissipation parameter increased the heat transfer at the wall while the magnetic dissipation parameter decreased it. The effect of small viscoelastic parameter in the presence of viscous dissipation and work due to deformation was examined by Cortell [9].

Knowledge of radiative heat transfer in industries is of great importance. Particularly, thermal radiation effect in power plant whose operation is at high temperature. Similarly, the relevance of heat source/sink in the management of electronic devices and systems cannot be ignored. To this end, Hayat et al. [10] studied the effect of thermal radiation in presence of viscous dissipation on second grade fluid. On the other hand, Bataller [11] examined the effects of heat generation/absorption, thermal radiation, viscous dissipation and work due to deformation over a viscoelastic stretching sheet.

Furthermore, Bhattacharyya et al. [12] investigated the effects of heat source/sink on the flow of two different classes of viscoelastic fluid (second grade and Walters Liquid B) over a linearly stretching sheet. Vajravelu and Roper [13] analysed the flow and heat transfer in a viscoelastic fluid over a stretching sheet with power law surface temperature in presence of dissipation function, internal heat generation/absorption and work due to deformation. The influence of space-temperature dependent heat source/sink in presence of heat radiation over porous stretching sheet was studied by Chauhan and Olkha [14]. Various models of the second grade fluid problem have been studied by other investigators such as Baris and Dokuz [15], Hayat et al. [16], Hameed et al. [17], Khan et al. [18], Akinbobola [19] and Makanda et al. [20].

The influences of temperature dependent viscosity on viscoelastic fluid such as second grade fluid causes changes in the properties of the fluid. For gases, the viscosity increases as temperature increases while for liquid it decreases as temperature increases. Consequently, a large amount of research work has been devoted to study the effects of many variable viscosity models. Massoudi and Phuoc [21] used Reynolds viscosity model to investigate the effect of variable viscosity in a fully developed flow of non-Newtonian fluid down a heated inclined plane. The same Reynolds law was used in generalised second grade fluid between two vertical parallel walls by Massoudi et al. [22]. Ramya et al. [23] studied the effects of temperature dependent viscosity on flow and heat transfer in a viscoelastic fluid in a porous medium. They assumed that the viscosity varies inversely as a function of temperature.

The property that describes the ability of a material to transfer heat is thermal conductivity and it plays a significant role in cooling. However, thickness and temperature can affect thermal conductivity. Thus, the influence of temperature dependent thermal conductivity in stretching sheet is of interest to researchers. Abel and Mahesha [24] examined the effects of variable thermal conductivity, non-uniform heat source and dissipative heat in the presence of thermal radiation on flow and heat transfer in viscoelastic fluid over a stretching sheet with an external magnetic field. Ahmad [25] considered a viscoelastic model over a stretching plate and heat transfer with variable thermal conductivity for two cases, namely: prescribed surface temperature and prescribed heat flux.

In view of the above research studies, it is evident that no analysis on the combined effects of temperature dependent viscosity and thermal conductivity in the presence of heat source/sink, viscous dissipation, work done due by deformation and thermal radiation in a second grade fluid flow has been carried out. Consequently, the main aim of the present work is to fill in this gap. We, therefore, derived the relevant partial differential equations for our model, employed similarity variables and obtained a coupled nonlinear ordinary differential equations which were solved numerically. Tables and graphs of pertinent parameters featured prominently in the investigation.

2. Flow analysis

An incompressible homogeneous fluid of second grade has a constitutive equation given by [26] as

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2(\mathbf{A}_1)^2. \tag{1}$$

Here, $\mathbf{T}$ is Cauchy stress tensor, P is pressure, $\mathbf{I}$ is the unit tensor, $\mu(\bar{T})$ is the temperature dependent dynamic viscosity, α_1 and α_2 are the material moduli while the kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as