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Motion around $L_{4,5}$ in the relativistic R3BP with smaller triaxial primary

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Abstract

This paper deals with the triangular points $L_{4,5}$ of the relativistic restricted three-body problem (R3BP) when the smaller primary is assumed triaxial. It is noticed that the locations and stability of the triangular points are affected by both relativistic and triaxiality pertubations. It can be easily seen that the range of stability region of these points is reduced by the effects of relativistic and triaxiality factors and more especially decreases with the increase of triaxiality factor.

Keywords: Celestial mechanics; Triaxiality; Relativity; R3BP

1. Introduction

The restricted three-body problem concerns the study of the motion of one infinitesimal celestial body in the gravitation field of two other bodies (conventionally called the primaries) moving along circular keplerian orbits around their center of mass. The third body has small mass with respect to others, and is treated like a test particle whose motion results determined by the two bodies, yet without affecting their motion in turn. It is an approximation of the three-body problem, which regards the study of the dynamics of three masses interacting by means of the gravitational force. This problem possesses five points called Lagrangian points, three of them are called collinear points L_1, L_2 and L_3 and are unstable, they lie on the line joining the primaries, the other two are called the triangular points L_4 and L_5 and are stable for the mass ratio $\mu \leq 0.038520\ldots$, Szebehely [1].

It is known that celestial bodies are irregular bodies and cannot be always considered as spherical in the restricted three-body problem, because the shape of the body affects the locations as well as the motion around equilibrium points. In most cases the planets and their natural satellites are extended bodies which are triaxial or oblate spheroids; this problem has wide applications in many astrophysical problems, Trojan asteroids, around the triangular points of

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the Sun-Jupiter system are examples of this. The lack of asphericity, triaxiality or oblateness of the celestial bodies causes large perturbations in a two-body orbit.

Some studies, which are related to the Lagrangian points by considering one or both primaries are oblate spheroids or triaxial, are discussed by SubbaRao and Sharma [2]; Sharma et al. [3]; Singh [4]; AbdulRaheem and Singh [5]; Sharma et al. [6], and Abouelmagd [7].

The theory of the general relativity is currently the most successful gravitational theory describing the nature of space and time, and well confirmed by observations [8]. For the application in celestial mechanics, the most important problem of general relativity is the problem of motion of material bodies.

For a test particle, the equations of motion are determined by the geodesic principle.

Brumberg [9,10] studied the problem in more details and collected most of the important results on relativistic celestial mechanics. He did not obtain only the equation of motion for the general problem of three bodies, but also deduced the equations of motion for the restricted problem of three bodies.

Bhatnagar and Hallan [11] studied the existence and linear stability of the triangular points $L_{4,5}$ in the relativistic R3BP, and found that $L_{4,5}$ are always unstable in the whole region $0 \leq \mu < \frac{1}{2}$ in contrast to the classical R3BP in which they are stable for $\mu < \mu_0$, where μ is the mass ratio and $\mu_0 = 0.038520\ldots$ is the Routh's value.

Douskos and Perdios [12] investigated the stability of the triangular points in the relativistic R3BP and contrary to the result of Bhatnagar and Hallan [11], they obtained a region of linear stability in the parameter space as $0 \leq \mu < \mu_0 - \frac{17\sqrt{69}}{486c^2}$ where $\mu_0 = 0.03852\ldots$ is Routh's value.

Katour et al. [13] obtained new locations of the triangular points in the framework of relativistic R3BP with oblateness and photo-gravitational corrections to triangular equilibrium points.

In the present work, we study the existence of the triangular points and their linear stability by considering the less massive primary as a triaxial rigid body.

This paper is organized as follows: In Section 2, the equations governing the motion are presented; Section 3 describes the positions of triangular points, while their linear stability is analyzed in Section 4; the obtained results are discussed in Section 5, finally Section 6 conveys the main findings of this paper.

2. Equations of motion

The pertinent equations of motion of an infinitesimal mass in the relativistic R3BP in a barycentric synodic coordinate system (ξ, η) and dimensionless variables with origin at the center of mass of the primaries can be written as Brumberg [9] and Bhatnagar and Hallan [11]:

$$\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= \frac{\partial W}{\partial \xi} - \frac{d}{dt}\left(\frac{\partial W}{\partial \dot{\xi}}\right) \\ \ddot{\eta} + 2n\dot{\xi} &= \frac{\partial W}{\partial \eta} - \frac{d}{dt}\left(\frac{\partial W}{\partial \dot{\eta}}\right) \end{aligned} \tag{1}$$

with

$$\begin{aligned} W = {} & \frac{1}{2}(\xi^2+\eta^2) + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} + \frac{1}{c^2}\Bigg[-\frac{3}{2}\left(1-\frac{1}{3}\mu(1-\mu)\right)(\xi^2+\eta^2) \\ & + \frac{1}{8}\left\{\dot{\xi}^2+\dot{\eta}^2+2(\xi\dot{\eta}-\eta\dot{\xi})+(\xi^2+\eta^2)\right\}^2 \\ & + \frac{3}{2}\left(\frac{1-\mu}{\rho_1}+\frac{\mu}{\rho_2}\right)\left(\dot{\xi}^2+\dot{\eta}^2+2(\xi\dot{\eta}-\eta\dot{\xi})+(\xi^2+\eta^2)\right) - \frac{1}{2}\left(\frac{(1-\mu)^2}{\rho_1^2}+\frac{\mu^2}{\rho_2^2}\right) \\ & + \mu(1-\mu)\left\{\left(4\dot{\eta}+\frac{7}{2}\xi\right)\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right) - \frac{\eta^2}{2}\left(\frac{\mu}{\rho_1^3}+\frac{1-\mu}{\rho_2^3}\right) + \left(\frac{-1}{\rho_1\rho_2}+\frac{3\mu-2}{2\rho_1}+\frac{1-3\mu}{2\rho_2}\right)\right\}\Bigg] \end{aligned} \tag{2}$$

$$n = 1 - \frac{3}{2c^2}\left(1-\frac{1}{3}\mu(1-\mu)\right) \tag{3}$$

$$\begin{aligned} \rho_1^2 &= (\xi+\mu)^2+\eta^2 \\ \rho_2^2 &= (\xi+\mu-1)^2+\eta^2 \end{aligned} \tag{4}$$