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Modeling and simulation of combustion fronts in porous media

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Abstract

We study a model for forward propagation of a combustion front through a porous medium. The reaction involves oxygen and a solid fuel. We assume that this solid fuel depends on the space variable. We also assume that the amount of gas produced by the reaction is equal to the amount of gas consumed by it. By actual solutions, we prove the existence and uniqueness of solution of the model. We show that temperature is non-decreasing function of time. We use the similarity variable to transform the system of partial differential equations, describing the problem under consideration, into a boundary value problem of coupled ordinary differential equations and an efficient numerical technique is implemented to solve the reduced system. The results are presented graphically and discussed. It is discovered that the heat transfer and species consumption are significantly influenced by the Frank–Kamenetskii number.

Keywords: Combustion front; Reaction–diffusion–convection system; Solid fuel; Porous media; Similarity solution

1. Introduction

Combustion is the exothermic oxidation of a fuel. In the case of a carbon-based compound, the products are primarily carbon dioxide, water and energy.

Combustion fronts in porous media have been studied by many authors during the last few decades. In particular, for combustion processes in oil recovery, models and results of numerical simulations have been presented. One of the first models of combustion in a petroleum reservoir was formulated by Gottfried [1]. The model consists of a system of six partial differential equations describing the flow of oil, water, and gas through the porous medium, together with a chemical reaction between oxygen and oil. Numerical simulations exhibit all of the main thermal and hydrodynamic features of in-situ combustion known from the laboratory, including propagation of the combustion zone, formation of a steam plateau, and formation of water and oil banks. Crookston and Culham [2] presented a general model for thermal recovery processes, as well as associated numerical procedures. In addition to the aspects of combustion processes modeled by Gottfried, they included such aspects as coke formation and oxidation.

These models are nonlinear reaction–diffusion–convection systems derived from the principle of conservation. In vector form in one space dimension, they have the form

$$H(U)_t + F(U)_x = (B(U)U_x)_x + G(U), \quad 0 < x < l,\ t > 0, \tag{1.1}$$

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Nomenclature

A	Arrhenius constant
c_{pg}	heat capacity of gas at constant pressure
E	activation energy
a	order of the gaseous reaction rate
R	gas constant
s	mass/weighted stoichiometric coefficients
T	temperature
t	time
x	position
C	concentration/mass fraction
D	diffusion coefficient
k	the flow resistance
Q	heat of reaction
v	seepage velocity
p	pressure

Greek Letters

λ	thermal conductivity
α	effective thermal diffusivity
ρ	density
ω	rate of coke consumption in the chemical reaction
θ	dimensionless temperature
$\in$	dimensionless activation energy $\left\{=\frac{RT_0}{E}\right\}$
δ	Frank–Kamenetskii number $\left\{=\frac{\alpha Q A\left(C_{ox}^0 p^* p'\right)^a C_f^0 {C_{ox}}'^a {C_f}' e^{-\frac{E}{RT_0}}}{\rho^* \rho' c_{pg} \in T_0 v'^2}\right\}$

Subscripts

sf	solid fuel
ox	oxygens
g	gas
f	fuel
0	initial
b	burner

Superscripts

i	injection

where l is the length of the porous medium and U is the vector of unknown quantities, such as temperature and densities. The first and second terms in (1.1) represent, respectively, accumulation and transport by convection of these quantities; the function G represents source terms due to chemical reactions and heat loss; and the term $(B(U)U_x)_x$ represents diffusion of heat, mass, etc. The combustion process is described by the solution of the system (1.1), with suitable initial and boundary conditions.

In the more recent literature, several authors have studied the oxidation of crude oil with air injected in porous media. These include Ayeni [3] who studied thermal runaway phenomena while investigating the reaction of oxygen and hydrogen. He provided useful theorems on such flows. Davies [4] who tracked an in-situ combustion front using thin flame technique. Schecter and Marchesin [5] who constructed a two-phase model for oxidation, involving air or