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On screw dislocation in a multiphase lamellar inclusion

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Abstract

The interaction of a screw dislocation in a lamellar inclusion with multiple boundaries is investigated. An analytical solution is obtained for the force acting on the dislocation, and equilibrium positions are established for physically interesting special cases, such as a double layer bounded by free plane surfaces. A functional relationship is obtained which expresses the force on a screw dislocation on one side of an interface in terms of the force on a screw dislocation on the opposite side of the interface, thereby contributing to reduction of effort in the calculation process.

Keywords: Screw dislocation; Multiphase; Lamellar inclusion

1. Introduction

Multiphase layered structures have important engineering applications, particularly in electronic and micro-electro-mechanical devices. The presence of defects, such as dislocations, can adversely affect the reliability of such devices [1]. Analytical solutions provide a cost-effective way of evaluating the influence of material parameters on the interaction energy and force acting on a dislocation in the presence of multiple boundaries, thereby contributing to an understanding of the mobility and trapping mechanism of dislocations.

Studies of dislocations in layered structures may be traced back to the pioneering work of Head [2] on the interaction of dislocations with boundaries. His classical solution for the force on a screw dislocation in an isotropic half space on which there is a surface film paved the way for similar investigations for a variety of configurations of layered materials (see, for example, [3–5]). More recently, variants of the image method have been used to study the interaction of dislocations with boundaries for some two-dimensional anti-plane configurations (see, for example, [6–10]). In [7], the technique of Mellin transform was used in conjunction with the image method to analyze the problem of two-dimensional dissimilar isotropic composite annular wedges subjected to anti-plane concentrated forces and screw dislocations. In [8], a generalized image method was developed and used to obtain solutions for a generalized dislocation in a solid consisting of three finite layers on a semi-infinite substrate. In [9], the method of image dislocations was used to obtain analytical solutions for the elastic fields due to an edge dislocation in a linearly elastic isotropic

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film–substrate, while in [10] a complex potential approach was employed in conjunction with the image method to study the interaction between an edge dislocation and an elastic thin layered semi-infinite matrix.

The objective of this work is to analyze the behavior of a screw dislocation in the interior of a layer of finite thickness within a multiphase laminate of isotropic elastic materials. Our strategy is to use image singularities of the infinite medium solution to construct the displacement fields in the phases such that the boundary conditions at the interfaces are satisfied. We shall obtain the force acting on the screw dislocation and use it to discuss the mobility and stability of the dislocation. The results of this investigation could throw more light on the solution for more general composite structures.

2. Formulation

The problem to be solved is that of a screw dislocation in the interior of a layer of finite thickness within a multiphase isotropic elastic material. The structure consists of five layers of equal thickness, a, sandwiched between two semi-infinite planes. A rectangular cartesian coordinate system (x_1, x_2, x_3) is assumed, and the seven phases of shear moduli μ_i, $(i = 1, 2, \ldots, 7)$ occupy the regions S_i defined by

$$\begin{aligned} &S_1 : x_1 > 0, \qquad S_7 : x_1 < -5a \\ &S_i : -ai < x_1 < -a(i-1), \quad (i = 1, \ldots, 5). \end{aligned} \tag{1}$$

It is assumed that the materials are perfectly bonded at the interfaces $x_1 = (1 - i)a$, where $i = 1, 2, \ldots, 6$. An infinitely long straight screw dislocation of Burger's vector $(0, 0, b)$ parallel to the x_3-axis is located at the point, $(-h, 0)$, in the second phase. Fig. 1 illustrates the configuration of the problem under consideration.

For this problem, the only non-vanishing component of the displacement field, $\overrightarrow{u} = (u, v, w)$, is the vertical component, w, and it is a function of x_1 and x_2. Consequently, the Navier displacement equilibrium equation of elastostatics,

$$(1 - 2\nu)\nabla^2\overrightarrow{u} + \nabla(\nabla \bullet \overrightarrow{u}) = 0, \tag{2}$$

reduces to the two-dimensional Laplace's equation

$$\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} = 0. \tag{3}$$

Let σ_{jk}, $(j, k = 1, 2, 3)$ denote the components of the stress field, where the subscripts 1, 2 and 3 stand for x_1, x_2 and x_3, respectively. Then, the antiplane shear stress components, σ_{3k} $(k = 1, 2)$, are the only non-vanishing components of the stress field for this problem. We denote by $w^{(i)}$ the vertical displacement in a phase of shear modulus μ_i while the corresponding stresses are $\sigma_{31}^{(i)}$ and $\sigma_{32}^{(i)}$. These quantities are related through the equations,

$$\sigma_{31} = \mu_i \frac{\partial w^{(i)}}{\partial x_1}, \qquad \sigma_{32} = \mu_i \frac{\partial w^{(i)}}{\partial x_2}. \tag{4}$$

The assumption of perfect bonding at the interface between adjacent phases of the composite structure implies that each $w^{(i)}$ must satisfy Eq. (3) subject to the perfect bond continuity conditions,

$$\begin{aligned} &w^{(i)} = w^{(i+1)}, \qquad \sigma_{31}^{i} = \sigma_{31}^{i+1}, \\ &\text{at} \quad x_i = (1 - i)a, \quad (i = 1, 2, \ldots, 6). \end{aligned} \tag{5}$$

In addition, the elastic fields must exhibit the singularity associated with a screw dislocation in the domain containing the dislocation and must vanish for $|x_1| \longrightarrow \infty$. Eqs. (3)–(5) determine the elastic fields in the seven-phase composite material under the influence of a screw dislocation.

3. Solution

Guided by the fundamental solution for a screw dislocation in an infinite plane, we shall construct the solution for the layered material by using an array of image singularities to account for the boundary conditions at the interfaces.