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On the generalization of the error and error estimation process of Ortiz's recursive formulation of the tau method

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Abstract

In this paper, the generalization of the Lanczos–Ortiz's Recursive formulation of the tau method for general non-overdetermined ordinary differential equations is presented. The generalization of the canonical polynomials and their derivatives for both overdetermined and non-overdetermined cases were reported in the earlier works of these authors, thus the emphasis here is on the error and the error estimate procedures. The accuracy of the results were established using some numerical examples and the induction principle.

Keywords: Error estimate; Canonical polynomials; Chebyshev polynomials; Tau approximant

Nomenclature

ϵ_i^* where $i = 0(1)m$ Error estimation term
ϵ Actual error term

1. Introduction

The introduction of canonical polynomials in the tau method by Lanczos (1938) with the limitation in application to first order differential equation was enhanced by Ortiz [6] when he generated these canonical polynomials recursively. Once the said limitations were removed, Aliabadi et al. [4] applied this variant in solving both moving and free boundary value problems. Using polynomial economization process of Lanczos [5], Adeniyi et al. [1] reported an efficient error estimation procedure, the extension of which was reported in Adeniyi et al. [3] to the solution of systems of first order differential equations. The quest for the automation of the recursive formulation of the tau method prompted the efforts at the generalization of the canonical polynomials for the non-overdetermined and overdetermined ordinary

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differential equations (see [7,8]). Therefore, in this work, before going to the generalization of both the approximant and the error estimation procedure, we shall restate the general formula for the canonical polynomials for the sake of completeness.

2. The general formula for the canonical polynomials and their derivatives for non-overdetermined ordinary differential equations

In [7,8] the class of mth order ODE (2.1) was considered

$$Ly(x) := \sum_{r=0}^{m}\left(\sum_{k=0}^{N_r} P_{r,k}x^k\right) y^{(r)}(x) = \sum_{r=0}^{F} f_r x^r \tag{2.1a}$$

$$L^* y(x_0) = y^{(k)}(x_0) = \alpha_k, \quad k = 1(1)m-1 \tag{2.1b}$$

where N_r, F are given non-negative integers and x_0, α_k, f_r, $P_{r,k}$ are given real numbers.

Definition 1. The ordinary differential equation (2.1a) is non-overdetermined if and only if N_r is less than or atmost equal to m (the order of the DE).

The general formula for the rth order canonical polynomial $Q_r(x)$ for the class of DE (2.1) is given by

$$Q_r(x) = \frac{x^r - \sum_{k=1}^{m}\sum_{j=k}^{m}\left(j!\binom{r}{j}P_{j,j-k}\right)Q_{r-k}(x)}{\sum_{k=0}^{m} k!\binom{r}{k}P_{k,k}}, \quad r \geq 0,\ j \leq r \tag{2.2}$$

and the associated derivative for the canonical polynomial is given by

$$Q_r^{(\lambda)}(x) = \frac{\lambda!\binom{r}{\lambda}x^{r-\lambda} - \sum_{k=1}^{m}\left(\sum_{j=k}^{m} j!\binom{r}{j}P_{j,j-k}\right)Q_{r-k}^{(\lambda)}(x)}{\sum_{k=0}^{m} k!\binom{r}{k}P_{k,k}}, \quad 0 \leq \lambda \leq r,\ j \leq r \tag{2.3}$$

where in (2.2) and (2.3) r is the order of the canonical polynomials, λ is the order of the derivatives and m is the order of the differential equation.

The canonical polynomials play a central role in the Ortiz recursive formulation of the tau method, thus the importance of (2.2) and (2.3) stated above.

3. The tau approximant

In this section, the tau approximant for the recursive formulation using (2.2) and (2.3) for the problem (2.1) is considered. We seek an approximant

$$y_n = \sum_{r=0}^{n} a_r x^r, \quad n < +\infty \tag{3.1}$$

which is the exact solution of the corresponding perturbed problem

$$Ly_n(x) = \sum_{r=0}^{F} f_r x^r + H_n(x) \tag{3.2}$$

where

$$H_n(x) = \sum_{k=1}^{m} \tau_k T_{n-k+1}^*(x) \tag{3.3}$$