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# On total labelings of graphs with prescribed weights

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**Abstract**

Let $G = (V, E)$ be a finite, simple and undirected graph. The edge-magic total or vertex-magic total labeling of $G$ is a bijection $f$ from $V(G) \cup E(G)$ onto the set of consecutive integers $\{1, 2, \ldots, |V(G)| + |E(G)|\}$, such that all the edge weights or vertex weights are equal to a constant, respectively. When all the edge weights or vertex weights are different then the labeling is called edge-antimagic or vertex-antimagic total, respectively.

In this paper we provide some classes of graphs that are simultaneously super edge-magic total and super vertex-antimagic total, that is, graphs admitting labeling that has both properties at the same time. We show several results for fans, sun graphs, caterpillars and prisms.


*Keywords:* Super edge-magic total labeling; Super vertex-antimagic total labeling; Total labeling

## 1. Introduction

We consider $G = (V, E)$ finite undirected graphs without loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$, where $n = |V(G)|$, $m = |E(G)|$. A *labeling* of a graph $G$ is any mapping that maps a certain set of graph elements to a certain set of positive integers. If the domain is the vertex (or edge) set, respectively, the labeling is called *vertex* (or *edge*) *labeling*, respectively. If the domain is both vertices and edges then the labeling is called a *total labeling*. The *edge weight* of an edge under the total labeling is the sum of the edge label and the labels of its end vertices. The *vertex weight* of a vertex under the total labeling is defined as sum of the vertex label itself and the labels of its incident edges.

A labeling is called *edge-magic total* (*vertex-magic total*) if the edge weights (vertex weights) are all the same. If the edge weights (vertex weights) are pairwise distinct then the total labeling is called *edge-antimagic total* (*vertex-antimagic total*). A graph that admits edge-magic total (edge-antimagic total) labeling or vertex-magic total (vertex-antimagic total) labeling is called an *edge-magic total* (*edge-antimagic total*) *graph* or *vertex-magic total* (*vertex-antimagic total*) *graph*, respectively.

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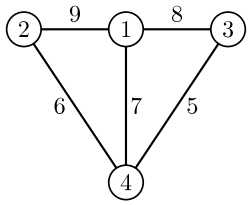


Fig. 1. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan $F_3$.

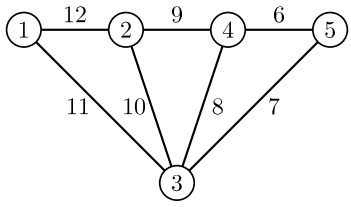


Fig. 2. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan $F_4$.

The subject of edge-magic total graph has its origin in the work of Kotzig and Rosa [1]. Vertex-magic total graphs are introduced by MacDougall, Miller, Slamin and Wallis in [2], see also [3,4]. The notation of edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller in [5] as a natural extension of magic valuation defined by Kotzig and Rosa in [1]. The vertex-antimagic total labeling of graphs was introduced in [6], see also [7]. If moreover the vertices are labeled with the smallest possible labels then, as is customary, the labeling is referred to as *super*. The concept of super-magic graphs was introduced by Stewart [8]. For further information on these types of labelings, one can see [9,7,10,11].

In [12] Bača, Miller, Phanalasy, Ryan, Semaničová-Feňovčíková and Sillasen proved that there exist some classes of stars, paths and cycle graphs which are simultaneously edge-magic total and vertex-antimagic total or simultaneously vertex-magic total and edge-antimagic total.

In this paper we will prove that some classes of fans, suns, caterpillars and prism graphs are simultaneously super edge-magic total and super vertex-antimagic total. For every of these graph we describe a total labeling that has both properties at the same time.

## 2. Fans, sun graphs, caterpillars and prisms

A *fan* $F_n$, $n \geq 2$, is a graph obtained by joining all vertices of path $P_n$ on $n$ vertices to another vertex, called center. The fan graph $F_n$ contains $n + 1$ vertices and $2n - 1$ edges.

**Theorem 1.** *The fan $F_n$ is simultaneously super edge-magic total and super vertex-antimagic total if and only if* $3 \leq n \leq 6$.

**Proof.** In [13], see also [7], is proved that the fan $F_n$ has a super edge-magic total labeling if and only if $2 \leq n \leq 6$. The fan $F_2$ is isomorphic to the cycle $C_3$. In [12] is showed that the cycle $C_3$ is not simultaneously super edge-magic total and super vertex-antimagic total.

For $3 \leq n \leq 6$ are the required labelings depicted in Figs. 1 through 4.

Fig. 1 depicts a simultaneously super edge-magic total and super vertex-antimagic total labeling for $F_3$ with edge weights equal 12 and vertex weights 16, 17, 22, 25.

A super edge-magic total labeling of the fan $F_4$ with edge weights 15 is illustrated in Fig. 2. This total labeling is also super vertex-antimagic with vertex weights 18, 24, 27, 33, 39.

Fig. 3 shows a simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan $F_5$ with edge weights 18 and with vertex weights 21, 29, 31, 38, 43, 57.

A total labeling of the fan $F_6$ with constant edge weights 21 is given in Fig. 4. This total labeling is also super vertex-antimagic with vertex weights 25, 34, 35, 39, 47, 52, 82. ■