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On graphs whose graphoidal domination number is one

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Abstract

Given a graph $G = (V, E)$, a set ψ of non-trivial paths, which are not necessarily open, called ψ-edges, is called a *graphoidal cover of* G if it satisfies the following conditions: $(GC - 1)$ Every vertex of G is an internal vertex of at most one path in ψ, and $(GC - 2)$ every edge of G is in exactly one path in ψ; the ordered pair (G, ψ) is called a *graphoidally covered graph*. Two vertices u and v of G are ψ*-adjacent* if they are the ends of an open ψ-edge. A set D of vertices in (G, ψ) is ψ*-dominating (in short* ψ*-dom set)* if every vertex of G is either in D or is ψ-adjacent to a vertex in D. Let $\gamma_\psi(G) = \inf\{|D| : Disa\psi - domsetofG\}$. A ψ-dom set D with $|D| = \gamma_\psi(G)$ is called a $\gamma_\psi(G)$-set. The *graphoidal domination number of a graph* G denoted by $\gamma_\psi^0(G)$ is defined as $inf\{\gamma_\psi(G) : \psi \in \mathcal{G}_G\}$. Let G be a connected graph with cyclomatic number $\mu(G) = (q - p + 1)$. In this paper, we characterize graphs for which there exists a non-trivial graphoidal cover ψ such that $\gamma_\psi(G) = 1$ and $l(P) > 1$ for each $P \in \psi$ and in this process we prove that the only such graphoidal covers are such that $l(P) = 2$ for each $P \in \psi$.
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Keywords: Graphoidal cover; Graphoidally covered graph; ψ-domination

1. Introduction

Throughout this paper, we shall follow the notation and terminology of Harary [1] and West [2] for graphs, unless specifically defined otherwise. Also, all graphs considered in this paper are simple, finite, undirected and connected. Let $G = (V, E)$ be a graph. The number of vertices and number of edges of G are denoted by p and q respectively. If $P = (u_0, u_1, u_2, \ldots, u_n)$ is a path, not necessarily open, in G then u_0 and u_n are called *external or end-vertices* of P and $u_1, u_2, \ldots, u_{n-1}$ are called *internal vertices* of P. Given a graph $G = (V, E)$, a set ψ of non-trivial paths, which are not necessarily open, called ψ*-edges*, is called a *graphoidal cover of* G if it satisfies the following conditions: $(GC - 1)$ Every vertex of G is an internal vertex of at most one path in ψ, and $(GC - 2)$ every edge of G is in exactly one path in ψ; the ordered pair (G, ψ) is called a *graphoidally covered graph*. The set of all graphoidal covers of a graph G is denoted by $\mathcal{G}_G$. For the study of graphoidal covers, the reader can refer to [3,4].

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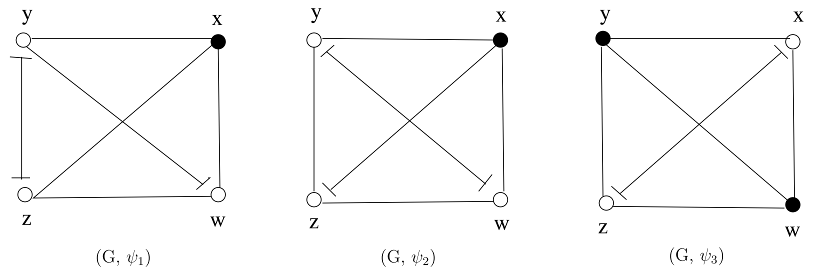

Fig. 1. $\psi_1 = \{(x, z, w, x), (x, y, w), (y, z)\}$, $\psi_2 = \{(x, z), (y, w), (x, y, z, w)\}$, $\psi_3 = \{(x, z), (y, w), (y, x, w), (y, z, w)\}$.

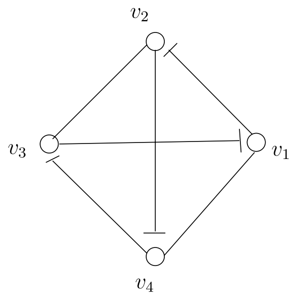

Fig. 2. $\psi = \{(v_1, v_3, v_2, v_4), (v_3, v_4, v_1, v_2)\}$.

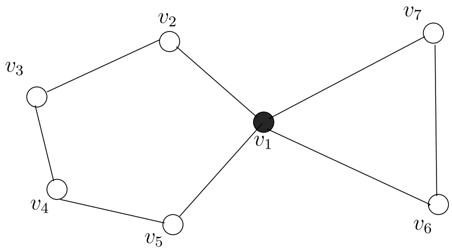

Fig. 3. $\psi_1 = \{(v_1, v_2, v_3, v_4, v_5, v_1), (v_1, v_6, v_7, v_1)\}$.

Fig. 1 exhibits a graphoidal cover ψ on a finite graph G where a black node denotes a vertex which is not internal to any ψ-edge, a small line segment attached to an end of an edge denotes an end vertex of the ψ-edge to which the edge belongs and a white vertex represents an internal vertex of the unique path to which it belongs.

Clearly, for any graph $G = (V, E)$, the set $E := E(G)$ of its edges (paths of length one) is itself a graphoidal cover of G, which we shall refer to as the *trivial graphoidal cover* of G. K_2 is the only graph having $\mathcal{G}_G = \{E(G)\}$. A graphoidal cover ψ of a graph containing at least one ψ-edge of length greater than one is called a *non-trivial graphoidal cover*. In Fig. 2, ψ is an example of a non-trivial graphoidal cover of the graph G.

The notion of domination in graphs is extended to graphoidal covers of a graph as follows. Two vertices u and v in a graphoidally covered graph (G, ψ) are said to be *ψ-adjacent* if they are the ends of an open ψ-edge. A set D of vertices in (G, ψ) is *ψ-dominating* (in short ψ-dom set) if every vertex of G is either in D or is ψ-adjacent to a vertex in D. We denote by $\mathcal{D}_\psi(G)$ the set of all ψ-dom sets of (G, ψ). Clearly the vertex set $V(G)$ of G is a ψ-dom set for any $\psi \in \mathcal{G}_G$ so that $V(G) \in \mathcal{D}_\psi(G)$ for any $\psi \in \mathcal{G}_G$; therefore, we call $V(G)$ the trivial ψ-dom set of (G, ψ). A graphoidal cover ψ such that $\mathcal{D}_\psi(G) = \{V(G)\}$ is called a *totally disconnecting graphoidal cover* of G in the sense that no two vertices in G are then ψ-adjacent, and the corresponding graphoidally covered graph (G, ψ) is said to be *ψ-independent*. Fig. 3 is an example of such a graphoidal cover of a graph. A ψ-dom set D of (G, ψ) is said to be *proper* if $V(G) - D \neq \emptyset$. Then $\gamma_\psi(G) = \inf\{|D| : D \in \mathcal{D}_\psi(G)\}$ is called *ψ-domination number of* G. A ψ-dom set D with $|D| = \gamma_\psi(G)$ is called a $\gamma_\psi(G)$-set. For $\psi = E(G)$, $\gamma_\psi(G)$ is the usual domination number of G, denoted by $\gamma(G)$.

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