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Radio mean labeling of a graph

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Abstract

A Radio Mean labeling of a connected graph G is a one to one map f from the vertex set $V(G)$ to the set of natural numbers N such that for two distinct vertices u and v of G, $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \operatorname{diam}(G)$. The radio mean number of f, $rmn(f)$, is the maximum number assigned to any vertex of G.The radio mean number of G, $rmn(G)$ is the minimum value of $rmn(f)$ taken over all radio mean labelings f of G. In this paper we find the radio mean number of graphs with diameter three, lotus inside a circle, Helms and Sunflower graphs.

Keywords: Radio labeling; Diameter; Wheel; Helms

1. Introduction

Throughout this paper we consider finite, simple, undirected and connected graphs. $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G. Also, for a graph G, p and q denote the number of vertices and edges respectively. In 2001, Chartrand et al. [1] defined the concept of radio labeling of G. Radio labeling of graphs is motivated by restrictions inherent in assigning channel frequencies for radio transmitters [1]. Radio labeling behavior of several graphs are studied by Kchikech et al. [2,3], Khennoufa et al. [4], Liu et al. [5–9], Van den Heuvel et al. [10] and Zhang [11]. Motivated by the radio labeling we define radio mean labeling of G. A radio mean labeling is a one to one mapping f from $V(G)$ to N satisfying the condition

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \operatorname{diam}(G) \tag{1.1}$$

for every $u, v \in V(G)$. The span of a labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G, $rmn(G)$ is the lowest span taken over all radio mean labelings of the graph G. The condition

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(1.1) is called radio mean condition. In this paper we determine the radio mean number of some graphs like graphs with diameter three, lotus inside a circle, gear graph, Helms and Sunflower graphs. Let x be any real number. Then $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms and definitions not defined here are followed from Harary [12] and Gallian [13].

2. Main results

Since any radio mean labeling f is one to one, it follows that $rmn(G) \geq |V(G)|$. Further if $\text{diam}(G) = d$ and $V(G) = \{v_1, v_2, \ldots, v_p\}$, then $f : V(G) \to \mathbb{N}$ defined by $f(v_i) = d + i - 2$, $1 \leq i \leq p$, is a radio mean labeling and hence $rmn(G) \leq p + d - 2$. In particular for any graph with $d = 2$, we have $rmn(G) = p$. Now if G is any graph with diameter 3 and if (u_1, u_2, u_3, u_4) is a diametrical path then f defined by $f(u_1) = 1$, $f(u_4) = 2$, $f(u_3) = 3$ and $f(v)$ for remaining vertices are arbitrarily assigned the labels $4, 5, \ldots, p$, then it can be easily verified that f is a radio mean labeling of G and hence $rmn(G) = p$. Hence the following problem naturally arises:

Problem 2.1. Characterize graphs G for which $rmn(G) = p$.

The following theorem gives another family of graphs G with $rmn(G) = p$.

The sunflower graph SF_n is obtained from a wheel with the central vertex v_0 and the cycle $C_n : v_1 v_2 \ldots v_n v_1$ and additional vertices $w_1 w_2 \ldots w_n$ where w_i is joined by edges to v_i, v_{i+1} where v_{i+1} is taken modulo n.

Theorem 2.1. *The radio mean number of the sunflower graph SF_n is its order.*

Proof. For $n \leq 5$, since $\text{diam}(SF_3) = 2$ and $\text{diam}(SF_4) = \text{diam}(SF_5) = 3$, the result follows. Assume $n \geq 6$. It is clear that $\text{diam}(SF_n) = 4$. Define the function f with co domain $\{1, 2, \ldots, 2n+1\}$ as follows: $f(w_1) = 1$, $f(w_2) = n$, $f(w_3) = 2$, $f(w_i) = i - 1$, $4 \leq i \leq n$, $f(v_0) = n + 1$ and $f(v_i) = n + 1 + i$, $1 \leq i \leq n$. We must show that the radio mean condition

$$d(u, v) + \left\lceil \frac{f(u) + f(v)}{2} \right\rceil \geq 5 \tag{2.1}$$

for every pair of vertices (u, v) where $u \neq v$.

Now, if either $f(u) \geq 6$ or $f(v) \geq 6$, then $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 4$ and hence (2.1) trivially holds. Hence let $1 \leq f(u)$, $f(v) \leq 5$. Clearly $u, v \in \{w_1, w_3, w_4, w_5, w_6\}$. If $u = w_i$ and $v = w_j$ and $|i - j| > 1$, then $d(u, v) = 3$ or 4 and $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 2$. Therefore (2.1) holds. If $u = w_i$, $v = w_{i+1}$, then $d(u, v) = 2$ and $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 3$. Hence (2.1) holds. □

The Helm H_n is obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle C_n.

Theorem 2.2. *The radio mean number of a Helm H_n is $2n + 1$.*

Proof. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1 u_2 \ldots u_n u_1$ and $V(K_1) = \{u_0\}$. Let w_i be the pendent vertex adjacent to u_i $(1 \leq i \leq n)$. Since $\text{diam}(H_3) = 3$, the result follows. Now let $n \geq 4$. Then $\text{diam}(H_n) = 4$. We define f on V as follows: $f(w_i) = i$ for all i with $1 \leq i \leq n$ and $f(u_i) = n + 1 + i$ for all i with $0 \leq i \leq n$.

Since $\text{diam}(H_n) = 4$, to prove that f is a radio mean labeling, we need to prove that

$$d(u, v) + \left\lceil \frac{f(u) + f(v)}{2} \right\rceil \geq 5 \tag{2.2}$$

for every pair of vertices (u, v) where $u \neq v$.

If either $f(u) \geq 6$ or $f(v) \geq 6$, then $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 4$ and hence (2.2) trivially holds. Hence let $1 \leq f(u), f(v) \leq 5$. If $n \geq 5$, it follows that $u, v \in \{w_1, w_2, w_3, w_4, w_5\}$. If $u = w_i$, $v = w_j$ and $|i - j| > 1$, then $d(u, v) = 4$ and (2.2) holds. If $u = w_i$ and $v = w_{i+1}$, then $d(u, v) = 3$ and $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 2$. Hence (2.2) holds. Thus f is a radio mean labeling of H_n. If $n = 4$, then $u, v \in \{w_1, w_2, w_3, w_4, u_0\}$ and since $f(u_0) = 5$, the inequality (2.2) holds. □