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Exhaustive fluid vacation model with Markov modulated load[☆]

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ABSTRACT

In this paper we analyze stable fluid vacation models with exhaustive discipline, in which the fluid source is modulated by a background continuous-time Markov chain and the fluid is removed at constant rate during the service period. Due to the continuous nature of the fluid the state space of the model becomes continuous, which is the major novelty and challenge of the analysis. We adapt the descendant set approach used in polling models to the fluid vacation model. We provide steady-state vector Laplace Transform and mean of the fluid level at arbitrary epoch. First we consider the case when the fluid input rate is less than the fluid service rate during service and later we study the case when the fluid input rate is larger than the fluid service rate in some states of the model.

1. Introduction

Fluid vacation model is an extension of the classical vacation model (see in [1,2]), in which fluid takes the role of the customer of the classical model. Due to the continuous nature of the fluid, the flow in and the removal of fluid are characterized by rates. Hence the state space becomes continuous, which is a challenge in the analysis comparing to that of the discrete state space of the classical vacation model. This requires different analysis techniques.

In this paper we investigate a fluid vacation model with exhaustive service when the fluid source is modulated by a background Markov chain. The main idea of the analysis is the extension of the descendant set approach (see in [3]) to the continuous fluid model context. This together with the transient analysis of the input fluid flow enables to describe the evolution of the joint fluid level and the state of the background Markov chain between the vacation end and vacation start epochs—on the Laplace transform (LT) level. The resulting relations are called the governing equations. From them we determine the steady-state probability vector of the background Markov chain at the vacation start epochs. In the course of the analysis we derive a relation for the steady-state vector LT and vector mean of the fluid level at arbitrary epoch in terms of the previously mentioned steady-state probability vector. We also derive the steady-state LT of the service time, which is the counterpart of the busy period analysis in the classical vacation queue.

This paper is an extended version of [4]. One of the two main additional contributions of the current work compared to [4] is the explicit expression for the embedded vector at service completion and the results which are built on that (e.g. Corollary 3), the second additional contribution is the extension of the model to the cases when the effective fluid

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rates (i.e., the fluid input rate minus the fluid service rate) can be positive during the service period. This extension makes the majority of the results obtained for the restricted model with strictly negative effective fluid rates invalid. We apply a new methodology based on the matrix analytic analysis of Markov fluid queues [5,6].

The rest of the paper is organized as follows. In Section 2 we present the fluid vacation model and the concept of embedding matrix LTs, which is needed for the extension of the descendant set approach to fluid model. In Section 3 we establish the governing equations of the model. The derivation of steady-state results follows in Section 4. Section 5 discusses the case with positive effective fluid rate during service and Section 6 presents numerical examples. Finally, Section 7 concludes the paper.

2. Model and notation

2.1. Model description

We consider a fluid vacation model with Markov modulated load and exhaustive discipline. The model has an infinite fluid buffer.

The input fluid flow of the buffer is determined by a modulating CTMC ($\Omega(t)$ for $t \geq 0$) with state space $\mathcal{S} = \{1, \ldots, L\}$ and generator $\mathbf{Q}$. When this Markov chain is in state j ($\Omega(t) = j$) then fluid flows to the buffer at rate r_j for $j \in \{1, \ldots, L\}$. We define the diagonal matrix $\mathbf{R} = \text{diag}(r_1, \ldots, r_L)$. During the service period the server removes fluid from the fluid buffer at finite rate $d > 0$. Consequently, when the overall Markov chain is in state j ($\Omega(t) = j$) then the fluid level of the buffer during the service period changes at rate $r_j - d$, otherwise during the vacation periods it changes at rate r_j, because there is no service. In the vacation model the length of the service period is determined by the applied discipline. In this work we consider the exhaustive discipline. Under exhaustive discipline the fluid is removed during the service period until the buffer becomes empty. Each time the buffer becomes empty the server takes a vacation period. During vacation periods there is no service thus the fluid level of the buffer is increasing by the actual flowing rates. The consecutive vacation times are independent and identically distributed (i.i.d.). The random variable of the vacation time, its probability distribution function (pdf), its Laplace transform (LT) and its mean are denoted by $\widetilde{\sigma}$, $\sigma(t) = \frac{d}{dt}Pr(\widetilde{\sigma} < t)$ and $\sigma^*(s) = E(e^{-s\widetilde{\sigma}})$, $\sigma = E(\widetilde{\sigma})$, respectively. We define the cycle time (or simply cycle) as the time between just after the starts of two consecutive service periods.

We set the following assumptions on the fluid vacation model:

- **A.1** The generator matrix $\mathbf{Q}$ of the modulating CTMC is irreducible.
- **A.2** The fluid rates are positive and finite, i.e. $r_j > 0$ for $j \in \{1, \ldots, L\}$.
- **A.3** The fluid level strictly decreases during the service period, i.e., $r_j < d$ for $j \in \{1, \ldots, L\}$.

Let $\boldsymbol{\pi}$ be the stationary probability vector of the modulating Markov chain. Due to assumption **A.1** the equations

$$\boldsymbol{\pi}\mathbf{Q} = 0, \qquad \boldsymbol{\pi}\mathbf{e} = 1. \tag{1}$$

uniquely determine $\boldsymbol{\pi}$, where $\mathbf{e}$ is the $L \times 1$ column vector of ones. The stationary fluid flow rate, λ, and the utilization ρ, is given as

$$\lambda = \boldsymbol{\pi}\mathbf{R}\mathbf{e}, \qquad \rho = \frac{\lambda}{d}, \tag{2}$$

respectively. The necessary condition of the stability of the fluid vacation model is that the mean fluid arrival rate $\lambda = \boldsymbol{\pi}\mathbf{R}\mathbf{e}$ is less than d, which is equivalent with $\rho < 1$.

If the amount of fluid served during a service period were limited, like e.g. in case of a model with time-limited discipline, then further restriction would be needed for the sufficiency. However the model with the exhaustive discipline does not have any load-independent limitation for a service period, therefore the above necessary condition is also a sufficient one for the stability of the system.

2.2. Notational conventions and embedded matrix LTs

For the i, jth element of the matrix $\mathbf{Z}$ the notations $\mathbf{Z}_{ij}$ or $[\mathbf{Z}]_{ij}$ are used. Similarly $\mathbf{z}_j$ and $[\mathbf{z}]_j$ denote the jth element of vector $\mathbf{z}$. When $\mathbf{X}^*(s)$, $Re(s) \geq 0$ is a matrix LT, $\mathbf{X}^{(k)}$ denotes its kth ($k \geq 1$) derivative at $s = 0$, i.e., $\mathbf{X}^{(k)} = \frac{d^k}{ds^k}\mathbf{X}^*(s)|_{s=0}$ and $\mathbf{X}^{(0)}$ denotes its value at $s = 0$, i.e., $\mathbf{X}^{(0)} = \mathbf{X}^*(0)$. Similar notations are applied for vector LT $\mathbf{x}^*(s)$ and scalar LT $x^*(s)$.

Let $\mathbf{Z}$ be an $L \times L$ rate matrix which has the following properties:

- the diagonal elements are negative ($\mathbf{Z}_{i,i} < 0$) and the other elements are non-negative ($\mathbf{Z}_{i,j} \geq 0$, for $i \neq j$),
- the row sums are zero.

For $Re(v) \geq 0$ let

$$\mathbf{H}(v) = \mathbf{D}v - \mathbf{Z}, \tag{3}$$

be a linear $L \times L$ matrix function of the complex variable v, where $\mathbf{Z}$ is a rate matrix and $\mathbf{D}$ is diagonal and its diagonal elements are positive, i.e. $[\mathbf{D}]_{j,j} > 0$ for $j \in \{1, \ldots, L\}$. That is $\mathbf{Z}$ and $\mathbf{D}$ are real. The matrix function $-\mathbf{H}(v)$ has the following properties: