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Equilibrium balking strategies in renewal input batch arrival queues with multiple and single working vacation

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ABSTRACT

This paper investigates equilibrium threshold balking strategies of customers in a renewal input batch arrival queue with multiple and single working vacation of the server. The vacation period, service period during normal service and vacation period are considered to be independent and exponentially distributed. Upon arriving, the customers decide whether to join or balk the queue based on observation of the system-length and status of the server. The waiting time in the system is associated with a linear cost–reward structure for estimating the net benefit if a customer wishes to participate in the system. Equilibrium customer strategy is studied under four cases: fully observable, almost observable, almost unobservable and fully unobservable. Using embedded Markov chain approach and system cost analysis, we obtain the equilibrium threshold. The analysis of unobservable cases is based on the roots of the characteristics equations formed using the probability generating function of embedded pre-arrival epoch probabilities. Equilibrium balking strategy may be useful in quality of service for EPON (ethernet passive optical network) as a multiple working vacation model and accounting through gatekeeper based H.323 protocols as a single working vacation model.

1. Introduction

Queueing systems where customers are allowed to depart before joining are often encountered in practical situation. In a reneging queueing system, the customers become impatient after joining the queue, that is, the customers may depart the system after the expiration of the patience timer. During the last few decades the economical aspect of queueing theory in view of customer's willingness to join the queue has gained a considerable amount of research thrust. Balking is different from reneging because the customers decide not to participate in a queueing system before joining the system. A reward–cost structure is assumed in the system which attracts the customer to join the queueing system. Each customer incurs a cost which is proportionate to the mean waiting time of the customer. Such kind of game theoretic perspective of queueing system was first studied by Naor [1] where an $M/M/1$ queueing model with linear cost–reward structure was considered. This study was extended by Edelson and Hildebrand [2] for unobservable case where the customers make their decisions without being informed about the system-length. The balking strategy was further extended by several authors in [3,4] for various kind of single server queueing system with features like reneging, retrials, etc. The results of these models are summarized in [5] with extensive bibliographical details. A vast literature survey on vacation queueing system can be

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found in Doshi [6], Takagi [7], Tian and Zhang (2006) [8] etc. Vacation queues may further be classified based on the number of vacations taken by the server. In case of multiple vacation queueing system the server keeps on taking vacations until at least a customer is found to be waiting at the end of a vacation. In case of single vacation system the server resumes service after completing a single vacation. If there are no customers found to be waiting at the end of single vacation, the server remains idle and waits for an arrival of a new customer at normal service mode, see Tian [9].

There are several papers in the literature for classical vacation queueing models with balking strategy. Burnetas and Economou [10] have considered an $M/M/1$ queue with setup times in which a setup time is incurred when the server starts a new busy period. Economou and Kanta [11–13] have made extensive analysis of balking with various kind of $M/M/1$ queue. An $M/M/1$ queue with compartmented waiting space is reported in [12]. In similar direction, an M/M/1 queue with unreliable server is analyzed in [11]. In recent times, Economou and Kanta [13] have also considered an $M/M/1$ queue with constant retrial policy. Balking strategy with setup/closedown times for an $M/M/1$ queue have been considered by Sun et al. [14] which is an enhancement over [10]. A balking strategy from reliability perspective for $M/M/1$ queue have been studied by Wang and Zhang [15] with unreliable server and delayed repair. Guo and Hassin [16] considered Markovian vacation queueing models with N-policy with and without the information about mean delay. Recently, Liu et al. [17] have considered equilibrium threshold strategies for observable queueing system with single vacation policy.

In recent times, queueing systems with working vacations in which the server works at a lower rate during vacations have been studied by several authors. Servi and Finn [18] first examined an $M/M/1$ multiple working vacation queue (known as $M/M/1/MWV$) in which the inter-arrival duration, service period and vacation period is considered to be independent and exponentially distributed. Baba [19] proposed multiple working vacations for $GI/M/1$ queueing system (known as $GI/M/1/MWV$) where the inter-arrival times form an independent identically distributed sequence of random variables (r.v.'s) having a general distribution function. The service period and vacation period are independent of arrival process. The service times during a normal service period, the service times during a working vacation, and working vacation times are exponentially distributed with lower rates. Along the same line, Banik et al. [20] have carried out the analysis for a finite buffer $GI/M/1/N$ queueing system. Interested readers can read Tian and Zhang [8] for more details. Li et al. [9] presented $GI/M/1$ queue with single working vacation. Zhang et al. [21] were among the first to study the equilibrium balking strategies for $M/M/1/MWV$ queueing systems.

In this paper, we investigate an equilibrium balking strategy for renewal input batch arrival queue for both the multiple and single working vacation policy (known as $GI^{[X]}/M/1/MWV$ and $GI^{[X]}/M/1/SWV$) queueing systems. Based on the availability of the information regarding system-length and server's status at the customer arrival epoch, four different cases have been classified. Namely, (1) Fully observable case: Customers are informed about the system-length as well as the server's status; (2) Almost observable case: Customers are informed only about the system-length; (3) Almost unobservable case: Customers are informed only about the server's status; (4) Fully unobservable case: Customers are not informed about the system-length nor the server's status. These assumptions are similar with [21]. In the study of single server working vacations, the existing literatures (for $GI/M/1/MWV$) [19] and (for $GI/M/1/SWV$) [9] focused on the matrix analytic method which is dependent on the structure of the transition probability matrix for analyzing renewal input batch arrival case. This motivates us to explore the feasibility of another method, namely the well known root finding method for renewal input batch arrival case with balking.

The contributions of the present paper, in our opinion, are consideration of equilibrium balking for renewal input batch-arrival queue with MWV and SWV models and proposing potential applications for both MWV and SWV models. Zhang et al. [21] analyzed an equilibrium balking for $M/M/1/MWV$ model. The roots method is used in our $GI^{[X]}/M/1/MWV$ and $GI^{[X]}/M/1/SWV$ models which is different from Zhang et al. [21]. In this context, readers are referred to [22–25] for a detailed analysis of root finding method. The rest of this paper is organized as follows. Section 2 defines both the models and balking strategy, Section 3 contains difference between SWV and MWV models. Sections 4 and 5 describes analytical framework of observable and unobservable queues, respectively. Section 6 analyzes the models at arbitrary epoch. Section 7 produces numerical results and Section 8 contains application towards voice/data transmission in EPON and Voice over Internet Protocol (VoIP) based network. Finally, conclusions and future scope are provided in Section 9.

2. Description of the models

Let us consider a single-server queueing system with partial batch rejection wherein customers arrive in batches of random size X with $P(X = i) = g_i (i \geq 1)$, mean $E(X) = \overline{g}$ and probability generating function (p.g.f.) $G(z)$. The inter-arrival times are independently and identically distributed (i.i.d.) random variables having general distribution $A(u) (u \geq 0)$, probability density function (p.d.f.) $h(u) (u \geq 0)$, Laplace–Stieltjes (LST) transform $a^*(s)$ and mean $1/\lambda$. The service discipline is first-come, first-served (FCFS). The service times during the busy period follow exponential distribution with parameter μ. The server commences a vacation of random length at the instant when the system becomes empty. A vacation time is exponentially distributed with parameter θ. If the customers are arriving in a vacation period, the server will serve them at a rate which is different from the rate of service during the service period. The service times during a working vacation also follow exponentially distributed with rate η. The server is allowed to take multiple working vacations as soon as the system becomes empty. We analyze both multiple working vacations and single working vacation models concurrently. Let ν be the indicator variable taking the value 0 for MWV model and 1 for SWV model.