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Global optimal solutions to general sensor network localization problem

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ABSTRACT

Sensor network localization problem is to determine the position of the sensor nodes in a network given pairwise distance measurements. Such problem can be formulated as a quartic polynomial minimization via the least squares method. This paper presents a canonical duality theory for solving this challenging problem. It is shown that the nonconvex minimization problem can be reformulated as a concave maximization dual problem over a convex set in a symmetrical matrix space, and hence can be solved efficiently by combining a general (linear or quadratic) perturbation technique with existing optimization techniques. Applications are illustrated by solving some relatively large-scale problems. Our results show that the general sensor network localization problem is not NP-hard unless its canonical dual problem has no solution in its positive definite domain. Fundamental ideas for solving general NP-hard problems are discussed.

1. Introduction

Sensor network localization is an important problem in communication and information theory, and has attracted an increasing attention [1–5]. The information collected through a sensor network can be interpreted and relayed far more effectively if it is known where the information is coming from and where it needs to be sent. Therefore, it is often very useful to know the positions of the sensor nodes in a network. Wireless sensor network consists of a large number of wireless sensors located in a geographical area with the ability to communicate with their neighbors within a limited radio range. Sensors collect the local environmental information, such as temperature or humidity, and can communicate with each other. Wireless sensor network is applicable to a range of monitoring applications in civil and military scenarios, such as geographical monitoring, smart homes, industrial control and traffic monitoring. There is an urgent need to develop robust and efficient algorithms that can identify sensor positions in a network by using only the measurements of the mutual distances of the wireless sensors from their neighbors, which is called neighboring distance measurements. The advance of wireless communication technology has made the sensor network a low-cost and highly efficient method for environmental observations.

Sensor network localization can also be formulated as an optimization problem by least squares method. However, this problem is nonconvex with many local minimizers. To find global optimal solutions by traditional theories and local-search methods is fundamentally difficult. It turns out that the general sensor localization problem has been considered to be NP-hard [6,7]. Several approximation methods have been developed for solving this difficult optimization problem (see [8] and

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references cited therein). The semi-definite programming (SDP) and second-order cone programming (SOCP) relaxations are two of the most popular methods studied recently [9–11]. The basic idea of SDP relaxation is to think of the quadratic terms as new variables subject to a linear matrix inequality. The SOCP relaxation is developed in a similar way. For both SDP and SOCP relaxation, computed sensor locations are not accurate when the solution of the localization problem is not unique. This is because many numerical schemes, such as primal–dual and interior point methods, for SDP or SOCP relaxation often return to the analytic center of the solution set. These solutions are, in general, not global optimal solutions.

Mathematically speaking, the localization problem in $\mathbb{R}^d$ can be stated as follows [12,13]: Consider a sensor network in $\mathbb{R}^d$ with m anchors and n sensors. An anchor is a node whose location $a_k \in \mathbb{R}^d$, where $k = 1, \ldots, m$, is known, and a sensor is a node whose location $x_i \in \mathbb{R}^d$, where $i = 1, \ldots, n$, is yet to be determined. For a pair of sensors x_i and x_j, their Euclidean distance is denoted as d_{ij}. Similarly, for a pair of sensor x_i and anchor a_k, their Euclidean distance is denoted as e_{ik}. In general, not all pairs of sensor /sensor and sensor/anchor are known, so the known pair-wise distances of sensor/sensor and sensor/anchor are denoted as $(i, j) \in \mathcal{A}_d$ and $(i, k) \in \mathcal{A}_e$, respectively. However, if we directly apply the general least squares method, the computation is very expensive and not practical for large problems [14].

Canonical duality theory developed from nonconvex analysis and global optimization (see [15,16]) is a powerful methodological theory, which has been used successfully for solving a large class of challenging problems in various disciplines, including global optimization [17], network communications [18], mathematical physics [19,20], neural network computations [21], material science [22], nonlinear dynamical systems [23], structural mechanics [24], and computational biology [25], etc. This paper presents an effective perturbation method based on the canonical duality theory to solve the general sensor network localization problem. Our main contribution is to show that this nonconvex optimization problem is not NP-hard unless its canonical dual problem has no solution. The rest of this paper is organized as follows. In the next section, we first reformulate the original problem as an optimization problem, where the decision variable is expressed in tensor (matrix) forms. In Section 3, the canonical duality theory is discussed in matrix space and a general analytical solution form is obtained by a complementary-dual principle. In Section 4, the general sensor localization problem is first reformulated in vector space and then transformed as a concave maximization dual problem over a convex feasible space $\mathcal{S}_a^+$. Based on the triality theory, a quadratic perturbation method is proposed, which shows that the nonconvex sensor network optimization problem is not NP-hard unless its canonical dual problem has no solution in $\mathcal{S}_a^+$. Section 5 presents some concrete numerical experiments for sensor localization problems with two, 18, 20 and 200 sensors. The cases with noise are also considered. Results are compared with standard semi-definite programming method. Concluding remarks are given in the last section.

The notations used in this paper are: $\mathbb{R}$ denotes the set of real numbers; A^T denotes the transpose of matrix A. For a finite set S, $|S|$ denotes its cardinality and the bilinear form $\langle u, u^* \rangle$ is simply the scalar product of two vectors or tensors.

2. Problem statement

Let us consider a general sensor network localization problem, where the sensor locations are to be determined by solving the system of nonlinear equations

$$(\mathcal{P}_0): \quad \|\mathbf{x}_i - \mathbf{x}_j\| = d_{ij}, \quad (i, j) \in \mathcal{A}_d, \tag{1}$$

$$\|\mathbf{x}_i - \mathbf{a}_k\| = e_{ik}, \quad (i, k) \in \mathcal{A}_e. \tag{2}$$

Here, the vectors $\mathbf{a}_k$, $k = 1, \ldots, m$, are specified anchors, where

$$\|\mathbf{x}_i - \mathbf{x}_j\| = \sqrt{\sum_{\alpha=1}^{d} (x_i^\alpha - x_j^\alpha)^2}$$

denotes the Euclidean distance between locations $\mathbf{x}_i$ and $\mathbf{x}_j \in \mathbb{R}^d$, $i = 1, \ldots, n; j = 1, \ldots, n$, and

$$\mathcal{A}_d = \{(i, j) \in [n] \times [n] \mid \|\mathbf{x}_i - \mathbf{x}_j\| = d_{ij},\ i < j,\ d_{ij} \text{ are given distances}\},$$

$$\mathcal{A}_e = \{(i, k) \in [n] \times [m] \mid \|\mathbf{x}_i - \mathbf{a}_k\| = e_{ik},\ e_{ik} \text{ are given distances}\},$$

where $[N] = \{1, \ldots, N\}$ for any integer N.

For a small number of sensors, it might be possible to compute sensor locations by solving Eqs. (1)–(2). However, solving this algebraic system can be very expensive computationally when the number of sensors is large.

By the least squares method [26], the sensor network localization problem $(\mathcal{P}_0)$ can be reformulated as a fourth-order polynomial optimization problem stated below.

$$(\mathcal{P}_1): \ \min \left\{ \Pi(\mathbf{X}) = \sum_{(i,j) \in \mathcal{A}_d} \frac{1}{2} w_{ij} (\|\mathbf{x}_i - \mathbf{x}_j\|^2 - d_{ij}^2)^2 + \sum_{(i,k) \in \mathcal{A}_e} \frac{1}{2} q_{ik} (\|\mathbf{x}_i - \mathbf{a}_k\|^2 - e_{ik}^2)^2 \right\}, \tag{3}$$

where $\mathbf{X} = [x_1, x_2, \ldots, x_n] = \{x_i^\alpha\} \in \mathbb{R}^{d \times n}$ is a matrix with each column x_i being a position in $\mathbb{R}^d$, $w_{ij}, q_{ik} > 0$ are given weights. Obviously, $\mathbf{X}$ are true sensor locations if and only if the optimal value is zero. This nonconvex optimization