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Research Problems from the BCC21

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ABSTRACT

A collection of open problems, mostly presented at the problem session of the 21st British Combinatorial Conference.

The Research Problems section presents unsolved problems in discrete mathematics. In special issues from conferences, most problems come from the meeting and are collected by the guest editors. In regular issues, the Research Problems collect problems submitted individually.

Older problems are acceptable if they are not widely known and the exposition features a new partial result. Concise definitions and commentary (such as motivation or known partial results) should be provided to make the problems accessible and interesting to a broad cross-section of the readership. Problems are solicited from all readers; they should be presented in the style below, occupy at most one journal page, and be sent to

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Most problems below were presented at the problem session of the 21st British Combinatorial Conference. Some problems contributed after the session were added. The problems are ordered according to subject matter.

Several of the problems presented at the meeting have been solved and hence have been removed, resulting in some gaps in the BCC numbering. One solved problem remains: it is a problem on on-line sorting proposed by Nicolas Lichiardopol whose solution by Adam Philpotts and Rob Waters appears in this volume.

These problems were collected and edited by:

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Comments and questions of a technical nature about a particular problem should be sent to the correspondent for that problem. Other comments and information about partial or full solutions should be sent to Professor Cameron (for potential later updates).

PROBLEM 950. (BCC21.1) A generalization of Erdős–Ko–Rado

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Let X be a set of cardinality n, and let r be a divisor of n. Partition X into n/r subsets $S_1, \ldots, S_{n/r}$, each of cardinality r.

Let $N(n, r, k)$ be the largest cardinality of a family $\mathcal{A}$ of k-subsets of X with the property that for all $A, B \in \mathcal{A}$, there exists $i \in \{1, \ldots, n/r\}$ such that $A \cap S_i \neq \emptyset$ and $B \cap S_i \neq \emptyset$.

Conjecture 1: Let $\mathcal{A}^*$ denote the family of all k-sets A such that $A \cap S_1 \neq \emptyset$. If $k < d_r n(1 - \epsilon_r(n))$, where $d_r = 1 - 2^{-1/r}$ and $\epsilon_r(n) = o(1)$, then

$$N(n, r, k) = |\mathcal{A}^*| = \binom{n}{k} - \binom{n-r}{k}.$$

Comment: For $r = 1$, the truth of the conjecture is the well-known Erdős–Ko–Rado Theorem [1]. The conjecture has also been proved for $r = 2$ (Johnson–Talbot, [2]). If n/r is odd and $k > d_r n(1+\delta_r(n))$, where $\delta_r(n) = o(1)$, then $N(n, r, k) = |\mathcal{A}_m|$, where

$$\mathcal{A}_m = \{A\colon\ A \text{ meets more than half of the sets } S_i\}.$$

A similar construction holds when n/r is even and k is in this range.

A more speculative question is open even for $r = 2$:

Question 2: Is it true that $N(n, r, k) = \max\{|\mathcal{A}^*|, |\mathcal{A}_m|\}$ for all k?

References

[1] P. Erdős, C. Ko, and R. Rado, Intersection theorems for systems of finite sets, *Quart. J. Math. Oxford* (2) **12** (1961), 313–320.
[2] J. R. Johnson and J. Talbot, *G*-intersection theorems for matchings and other graphs, *Combin. Probab. Comput.* **17** (2008), 559–575.

PROBLEM 951. (BCC21.2) A-optimality of graphs

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Given positive integers v and b such that $b \geq v - 1$, a connected graph with v vertices and b edges is *A-optimal* if it minimizes the total variance among all graphs with v vertices and b edges. Regarding the graph as an electrical network with 1 Ω resistors on every edge, A-optimality is equivalent to minimizing the sum of the resistances of the network between all pairs of terminals.

Concerning leaves in A-optimal graphs, the following properties are known (see [1]):
(a) if $b \geq v(v-1)/2$ then A-optimal graphs have no leaves;
(b) for each $c \geq 0$, there is a threshold $N(c)$ such that if $b = v + c$ and $v \geq N(c)$, then A-optimal graphs have many leaves.

Question: Does there exist a threshold function f such that

1. if $b > f(v)$, then A-optimal graphs have no leaves, and
2. if $b \leq f(v)$, then A-optimal graphs have (many) leaves?

If such a function exists, find it (explicitly or asymptotically).

Reference

[1] R. A. Bailey, Designs for two-colour microarray experiments, *J. Roy. Statist. Soc. Ser. C* **56** (2007), 365–394.

PROBLEM 952. (BCC21.3) The row space of an adjacency matrix

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Let G be a graph with at least one edge, and let A be the adjacency matrix of G.

Question: Is it always true that there is a nonzero $\{0, 1\}$-vector in the row space of A (over the real numbers) that is not a row of A?