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CT bursts—from classical to array coding

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Abstract

R.T. Chien and D.T. Tang [On definition of a burst, IBM J. Res. Develop. 9 (1965) 292–293] introduced the concept of Chien and Tang bursts (CT bursts) for classical coding systems where codes are subsets (or subspaces) of the space F_q^n, the space of all n-tuples with entries from a finite field F_q. In this paper, we extend the notion of CT bursts for array coding systems where array codes are subsets (or subspaces) of the space $\mathrm{Mat}_{m\times s}(F_q)$, the linear space of all $m \times s$ matrices with entries from a finite field F_q, endowed with a non-Hamming metric [M.Yu. Rosenbloom, M.A. Tsfasman, Codes for m-metric, Problems Inform. Transmission 33 (1997) 45–52]. We also obtain some bounds on the parameters of array codes for the detection and correction of CT burst errors.

MSC: 94B05

Keywords: CT bursts; Linear codes; Array codes

1. Introduction

In a classical coding setting [3,7,12], codes are subsets (or subspaces) of ambient space F_q^n and are investigated with respect to the Hamming metric. Also, array codes having 2-dimensional arrays as code vectors have been studied by many authors [2,8,17] etc. Recently in [14], m-metric array codes which are subsets (or subspaces) of linear space of all $m \times s$ matrices $\mathrm{Mat}_{m\times s}(F_q)$ with entries from a finite field F_q endowed with a non-Hamming metric were introduced and some bounds on code parameters were obtained. This newly defined non-Hamming metric gained attention of several mathematicians as a result of which there has been a recent growth of interest and research in m-metric array codes (e.g. [5,6,9–11,15,16]).

Here is a model of an information transmission for which array coding is useful and the non-Hamming metric defined in [14] is the natural quality characteristic of a code. Suppose that a sender transmits messages, each being an s-tuple of m-tuples of q-ary symbols over m parallel channels. We assume that there is an interfering noise in the channels which creates errors in the transmitted message. An important and practical situation is when errors are not scattered randomly in the code array (or code matrix) but are in cluster form and are confined to a submatrix part of the code matrix. These errors arise, for example, due to lightning and thunder in deep space and satellite communications. Motivated by this idea, the author introduced the notion of bursts [9] in array coding. In this paper, we introduce the class of Chien and

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Tang bursts (CT bursts) in array coding which is a superclass of bursts considered in [9] and is also a generalization of CT bursts introduced by Chien and Tang [4] for classical coding systems. We also obtain some bounds on the parameters of m-metric array codes for the detection and correction of CT bursts. The study of CT bursts is important due to the fact that the number of CT bursts of a particular order is greater than the number of usual bursts [9] of the same order and the code which can correct CT bursts can also correct all usual bursts of the same order. Also, CT bursts have been found useful in error analysis experiment on telephone lines [1].

2. Definitions and notations

Let F_q be a finite field of q elements. Let $\mathrm{Mat}_{m\times s}(F_q)$ denote the linear space of all $m \times s$ matrices with entries from F_q. An m-metric array code is a subset of $\mathrm{Mat}_{m\times s}(F_q)$ and a linear m-metric array code is an F_q-linear subspace of $\mathrm{Mat}_{m\times s}(F_q)$. Note that the space $\mathrm{Mat}_{m\times s}(F_q)$ is identifiable with the space F_q^{ms}. Every matrix in $\mathrm{Mat}_{m\times s}(F_q)$ can be represented as a $1 \times ms$ vector by writing the first row of matrix followed by second row and so on. Similarly, every vector in F_q^{ms} can be represented as an $m \times s$ matrix in $\mathrm{Mat}_{m\times s}(F_q)$ by separating the coordinates of the vector into m groups of s-coordinates.

There are two equivalent ways of defining the non-Hamming weight and metric on the space $\mathrm{Mat}_{m\times s}(F_q)$, viz. row weight and column weight [6,14]. We consider the row weight definition which runs as follows:

Let $Y \in \mathrm{Mat}_{1\times s}(F_q)$ with $Y = (y_1, y_2, \ldots, y_s)$. Define row weight (or weight) of Y as

$$\mathrm{wt}_\rho(Y) = \begin{cases} \max\{i \mid y_i \neq 0\} & \text{if } Y \neq 0, \\ 0 & \text{if } Y = 0. \end{cases}$$

We extend the definitions of wt_ρ to the class of $m \times s$ matrices as

$$\mathrm{wt}_\rho(A) = \sum_{i=1}^{m} \mathrm{wt}_\rho(R_i),$$

where

$$A = \begin{bmatrix} R_1 \\ R_2 \\ \cdots \\ R_m \end{bmatrix} \in \mathrm{Mat}_{m\times s}(F_q)$$

and R_i denotes the ith row of A. Then wt_ρ satisfies $0 \leqslant \mathrm{wt}_\rho(A) \leqslant n \ (=ms) \forall A \in \mathrm{Mat}_{m\times s}(F_q)$ and determines a metric on $\mathrm{Mat}_{m\times s}(F_q)$ known as row-metric or m-metric or RT metric.

In this paper, we take distance and weight in the sense of row-metric.

3. CT bursts in m-metric array codes

We now define CT bursts in m-metric array codes:

Definition 3.1. A CT burst of order pr (or $p \times r$) $(1 \leqslant p \leqslant m, 1 \leqslant r \leqslant s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is an $m \times s$ matrix in which all the non-zero entries are confined to some $p \times r$ submatrix which has non-zero first row and first column.

Observations. (1) For $m = p = 1$, Definition 3.1 reduces to the definition of CT burst for classical codes [4].

(2) The class of usual bursts [9] in $\mathrm{Mat}_{m\times s}(F_q)$ is a subclass of the class of CT bursts.

(3) It is clear that the non-zero $p \times r$ submatrix in a CT burst of order pr $(1 \leqslant p \leqslant m, 1 \leqslant r \leqslant s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ can have (i, j) as its starting position where $1 \leqslant i \leqslant m - p + 1$ and $1 \leqslant j \leqslant s - r + 1$.

Remark 3.1. A CT burst of order pr or less $(1 \leqslant p \leqslant m, 1 \leqslant r \leqslant s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is a CT burst of order cd (or $c \times d$) where $1 \leqslant c \leqslant p \leqslant m$ and $1 \leqslant d \leqslant r \leqslant s$.